In a time T (as just defined), the first wave crest has moved a distance $d = v_{snd}T = \lambda$, where v_{snd} is the velocity of the sound wave in air (which is the same whether the source is moving or not). In this same time, the source has moved a distance $d_{source} = v_{source}T$. Then the distance between successive wave crests, which is the wavelength λ' the observer will perceive, is

Frequency change, moving source, fixed observer

$$\begin{aligned}\lambda' &= d - d_{source}\\ &= \lambda - v_{source}T\\ &= \lambda - v_{source}\frac{\lambda}{v_{snd}}\\ &= \lambda\left(1 - \frac{v_{source}}{v_{snd}}\right).\end{aligned}$$

We subtract λ from both sides of this equation and find that the shift in wavelength, $\Delta\lambda$, is

$$\Delta\lambda = \lambda' - \lambda = -\lambda\frac{v_{source}}{v_{snd}}.$$

So the shift in wavelength is directly proportional to the source speed v_{source}. The frequency f' that will be perceived by our stationary observer on the ground is given by (Eq. 11–12)

$$f' = \frac{v_{snd}}{\lambda'} = \frac{v_{snd}}{\lambda\left(1 - \frac{v_{source}}{v_{snd}}\right)}.$$

Since $v_{snd}/\lambda = f$, then

$$f' = \frac{f}{\left(1 - \frac{v_{source}}{v_{snd}}\right)}. \qquad \begin{bmatrix}\text{source moving toward}\\ \text{stationary observer}\end{bmatrix} \quad \textbf{(12–2a)}$$

Because the denominator is less than 1, the observed frequency f' is greater than the source frequency f. That is, $f' > f$. For example, if a source emits a sound of frequency 400 Hz when at rest, then when the source moves toward a fixed observer with a speed of 30 m/s, the observer hears a frequency (at 20°C) of

$$f' = \frac{400\text{ Hz}}{1 - \frac{30\text{ m/s}}{343\text{ m/s}}} = 438\text{ Hz}.$$

Now consider a source moving *away* from the stationary observer at a speed v_{source}. Using the same arguments as above, the wavelength λ' perceived by our observer will have the minus sign on d_{source} (at top of this page) changed to plus:

$$\begin{aligned}\lambda' &= d + d_{source}\\ &= \lambda\left(1 + \frac{v_{source}}{v_{snd}}\right).\end{aligned}$$

The difference between the observed and emitted wavelengths will be $\Delta\lambda = \lambda' - \lambda = +\lambda(v_{source}/v_{snd})$. The observed frequency of the wave, $f' = v_{snd}/\lambda'$, will be

$$f' = \frac{f}{\left(1 + \frac{v_{source}}{v_{snd}}\right)}. \qquad \begin{bmatrix}\text{source moving away from}\\ \text{stationary observer}\end{bmatrix} \quad \textbf{(12–2b)}$$

If a source emitting at 400 Hz is moving away from a fixed observer at 30 m/s, the observer hears a frequency $f' = (400\text{ Hz})/[1 + (30\text{ m/s})/(343\text{ m/s})] = 368\text{ Hz}$.