

FIGURE 12-7 Standing waves on a string—only the lowest three frequencies are shown.

PHYSICS APPLIED

Stringed Instruments

We saw in Chapter 11, Fig. 11–40, how standing waves are established on a string, and we show this again here in Fig. 12–7. Such standing waves are the basis for all stringed instruments. The pitch is normally determined by the lowest resonant frequency, the **fundamental**, which corresponds to nodes occurring only at the ends. The string vibrating up and down as a whole corresponds to a half wavelength as shown at the top of Fig. 12–7; so the wavelength of the fundamental on the string is equal to twice the length of the string. Therefore, the fundamental frequency is $f_1 = v/\lambda = v/2L$, where v is the velocity of the wave on the string. The possible frequencies for standing waves on a stretched string are whole-number multiples of the fundamental frequency:

$$f_n = nf_1 = n\frac{v}{2L}, \qquad n = 1, 2, 3, \dots$$

(just as in Eq. 11–19b), where n=1 refers to the fundamental and $n=2,3,\cdots$ are the overtones. All of the standing waves, $n=1,2,3,\cdots$, are called harmonics[†], as we saw in Section 11–13.

When a finger is placed on the string of a guitar or violin, the effective length of the string is shortened. So its fundamental frequency, and pitch, is higher since the wavelength of the fundamental is shorter (Fig. 12–8). The strings on a guitar or violin are all the same length. They sound at a different pitch because the strings have different mass per unit length, m/L, which affects the velocity as seen in Eq. 11–13,

$$v = \sqrt{F_{\rm T}/(m/L)}$$
. [stretched string]

Thus the velocity on a heavier string is less and the frequency will be less for the same wavelength. The tension F_T may also be different. Adjusting the tension is the means for tuning the pitch of each string. In pianos and harps, on the other hand, the strings are of different lengths. For the lower notes the strings are not only longer, but heavier as well, and the reason is illustrated in the following Example.

EXAMPLE 12-7 Piano strings. The highest key on a piano corresponds to a frequency about 150 times that of the lowest key. If the string for the highest note is 5.0 cm long, how long would the string for the lowest note have to be if it had the same mass per unit length and was under the same tension?

APPROACH Since $v = \sqrt{F_T/(m/L)}$, the velocity would be the same on each string. So the frequency is inversely proportional to the length L of the string $(f = v/\lambda = v/2L)$.

[†]When the resonant frequencies above the fundamental (that is, the overtones) are integral multiples of the fundamental, as here, they are called harmonics. But if the overtones are not integral multiples of the fundamental, as is the case for a vibrating drumhead, for example, they are not harmonics.

FIGURE 12–8 The wavelength of (a) an unfingered string is longer than that of (b) a fingered string. Hence, the frequency of the fingered string is higher. Only one string is shown on this guitar, and only the simplest standing wave, the fundamental, is shown.

