Standing waves are produced not only on strings, but on any object that is struck, such as a drum membrane or an object made of metal or wood. The resonant frequencies depend on the dimensions of the object, just as for a string they depend on its length. Large objects have lower resonant frequencies than small objects. All musical instruments, from stringed instruments to wind instruments (in which a column of air vibrates as a standing wave) to drums and other percussion instruments, depend on standing waves to produce their musical sounds, as we shall see in Chapter 12.
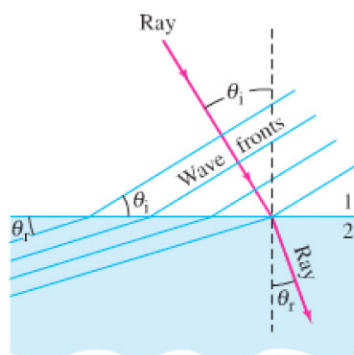
* 11–14 Refraction[†]

When any wave strikes a boundary, some of the energy is reflected and some is transmitted or absorbed. When a two- or three-dimensional wave traveling in one medium crosses a boundary into a medium where its speed is different, the transmitted wave may move in a different direction than the incident wave, as shown in Fig. 11–41. This phenomenon is known as **refraction**. One example is a water wave; the velocity decreases in shallow water and the waves refract, as shown in Fig. 11–42 below. [When the wave velocity changes gradually, as in Fig. 11–42, without a sharp boundary, the waves change direction (refract) gradually.]

In Fig. 11–41, the velocity of the wave in medium 2 is less than in medium 1. In this case, the wave front bends so it travels more nearly parallel to the boundary. That is, the *angle of refraction*, θ_r, is less than the *angle of incidence*, θ_i. To see why this is so, and to help us get a quantitative relation between θ_r and θ_i, let us think of each wave front as a row of soldiers. The soldiers are marching from firm ground (medium 1) into mud (medium 2) and hence are slowed down after the boundary. The soldiers that reach the mud first are slowed down first, and the row bends as shown in Fig. 11–43a. Let us consider the wave front (or row of soldiers) labeled A in Fig. 11–43b. In the same time t that A_1 moves a distance $l_1 = v_1 t$, we see that A_2 moves a distance $l_2 = v_2 t$. The two right triangles in Fig. 11–43b, shaded yellow and green, have the side labeled a in common. Thus

$$\sin \theta_1 = \frac{l_1}{a} = \frac{v_1 t}{a}$$

since a is the hypotenuse, and

$$\sin \theta_2 = \frac{l_2}{a} = \frac{v_2 t}{a}.$$

Dividing these two equations, we obtain the *law of refraction*:

Law of refraction

$$\frac{\sin \theta_2}{\sin \theta_1} = \frac{v_2}{v_1}. \qquad (11\text{–}20)$$

Since θ_1 is the angle of incidence (θ_i), and θ_2 is the angle of refraction (θ_r), Eq. 11–20 gives the quantitative relation between the two. If the wave were

[†] This Section and the next are covered in more detail in Chapters 23 to 25, on optics.

FIGURE 11–41 Refraction of waves passing a boundary.

FIGURE 11–42 Water waves refract gradually as they approach the shore, as their velocity decreases. There is no distinct boundary, as in Fig. 11–41, because the wave velocity changes gradually.