

The amplitude of a wave also decreases with distance. Since the intensity is proportional to the square of the amplitude (Eq. 11-15), the amplitude A must decrease as $1/r$ so that $I \propto A^2$ will be proportional to $1/r^2$ (as in Eq. 11-16b). Hence

$$A \propto \frac{1}{r}.$$

If we consider again two distances from the source, r_1 and r_2 , then

$$\frac{A_2}{A_1} = \frac{r_1}{r_2}.$$

When the wave is twice as far from the source, the amplitude is half as large, and so on (ignoring damping due to friction).

EXAMPLE 11-13 Earthquake intensity. The intensity of an earthquake P wave traveling through the Earth and detected 100 km from the source is $1.0 \times 10^6 \text{ W/m}^2$. What is the intensity of that wave if detected 400 km from the source?

APPROACH We assume the wave is spherical, so the intensity decreases as the square of the distance from the source.

SOLUTION At 400 km the distance is 4 times greater than at 100 km, so the intensity will be $(\frac{1}{4})^2 = \frac{1}{16}$ of its value at 100 km, or $(1.0 \times 10^6 \text{ W/m}^2)/16 = 6.3 \times 10^4 \text{ W/m}^2$.

NOTE Using Eq. 11-16c directly gives:

$$I_2 = I_1 r_1^2 / r_2^2 = (1.0 \times 10^6 \text{ W/m}^2)(100 \text{ km})^2 / (400 \text{ km})^2 = 6.3 \times 10^4 \text{ W/m}^2.$$

The situation is different for a one-dimensional wave, such as a transverse wave on a string or a longitudinal wave pulse traveling down a thin uniform metal rod. The area remains constant, so the amplitude A also remains constant (ignoring friction). Thus the amplitude and the intensity do not decrease with distance.

In practice, frictional damping is generally present, and some of the energy is transformed into thermal energy. Thus the amplitude and intensity of a one-dimensional wave will decrease with distance from the source. For a three-dimensional wave, the decrease will be greater than that discussed above, although the effect may often be small.

* 11-10 Intensity Related to Amplitude and Frequency

We can obtain an explicit relation between the energy carried by a wave, or the wave's intensity I , and the amplitude and frequency of the wave. For a sinusoidal wave of frequency f , the particles move in SHM as a wave passes, so each particle has an energy $E = \frac{1}{2}kA^2$, where A is the amplitude of its motion, either transversely or longitudinally. Using Eq. 11-7b, we can write k in terms of the frequency: $k = 4\pi^2mf^2$, where m is the mass of a particle (or small volume) of the medium. Then

$$E = \frac{1}{2}kA^2 = 2\pi^2mf^2A^2.$$

The mass $m = \rho V$, where ρ is the density of the medium and V the volume of a small slice of the medium as shown in Fig. 11-31. The volume $V = Sl$, where S is the cross-sectional surface area through which the wave travels. (We use S instead of A for area because we are using A for amplitude.) We can write l as the distance the wave travels in a time t as $l = vt$, where v is the speed of the wave. Thus $m = \rho V = \rho Sl = \rho Svt$, and

$$E = 2\pi^2\rho Svt f^2 A^2. \quad (11-17a)$$

From this equation, we see again the important result that the energy transported by a wave is proportional to the square of the amplitude. The power

FIGURE 11-31 Calculating the energy carried by a wave moving with velocity v .

