11–6 Forced Vibrations; Resonance

When a vibrating system is set into motion, it vibrates at its natural frequency (Eqs. 11–7b and 11–11b). However, a system may have an external force applied to it that has its own particular frequency. Then we have a **forced vibration**. For example, we might pull the mass on the spring of Fig. 11–1 back and forth at an externally applied frequency f. The mass then vibrates at the external frequency f of the external force, even if this frequency is different from the **natural frequency** of the spring, which we will now denote by f_0, where (see Eq. 11–7b)

$$f_0 = \frac{1}{2\pi} \sqrt{\frac{k}{m}}.$$

For a forced vibration, the amplitude of vibration is found to depend on the difference between f and f_0, and is a maximum when the frequency of the external force equals the natural frequency of the system—that is, when $f = f_0$. The amplitude is plotted in Fig. 11–18 as a function of the external frequency f. Curve A represents light damping and curve B heavy damping. The amplitude can become large when the external driving frequency f is near the natural frequency, $f \approx f_0$, as long as the damping is not too large. When the damping is small, the increase in amplitude near $f = f_0$ is very large (and often dramatic). This effect is known as **resonance**. The natural vibrating frequency f_0 of a system is also called its **resonant frequency**.

A simple illustration of resonance is pushing a child on a swing. A swing, like any pendulum, has a natural frequency of oscillation. If you push on the swing at a random frequency, the swing bounces around and reaches no great amplitude. But if you push with a frequency equal to the natural frequency of the swing, the amplitude increases greatly. At resonance, relatively little effort is required to obtain a large amplitude.

The great tenor Enrico Caruso was said to be able to shatter a crystal goblet by singing a note of just the right frequency at full voice. This is an example of resonance, for the sound waves emitted by the voice act as a forced vibration on the glass. At resonance, the resulting vibration of the goblet may be large enough in amplitude that the glass exceeds its elastic limit and breaks.

Since material objects are, in general, elastic, resonance is an important phenomenon in a variety of situations. It is particularly important in building, although the effects are not always foreseen. For example, it has been reported that a railway bridge collapsed because a nick in one of the wheels of a crossing train set up a resonant vibration in the bridge. Marching soldiers break step when crossing a bridge to avoid the possibility that their rhythmic march might match a resonant frequency of the bridge. The collapse of the Tacoma Narrows Bridge (Fig. 11–19a) in 1940 occurred as a result of gusting winds whose approximate frequency matched that of a natural frequency of the bridge, thus driving the span into large-amplitude oscillatory motion. Bridges and tall buildings are now designed with more inherent damping. The Oakland freeway collapse in the 1989 California earthquake (Fig. 11–19b) involved resonant oscillation of a section built on mudfill.

Resonance can be very useful, too, and we will meet important examples later, such as in musical instruments and tuning a radio. We will also see that vibrating objects often have not one, but many resonant frequencies.
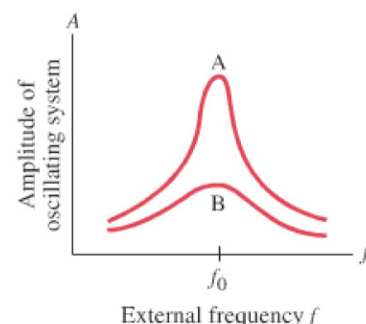
FIGURE 11–18 Resonance for lightly damped (A) and heavily damped (B) systems.

PHYSICS APPLIED
Swinging

PHYSICS APPLIED
Shattering glass via resonance

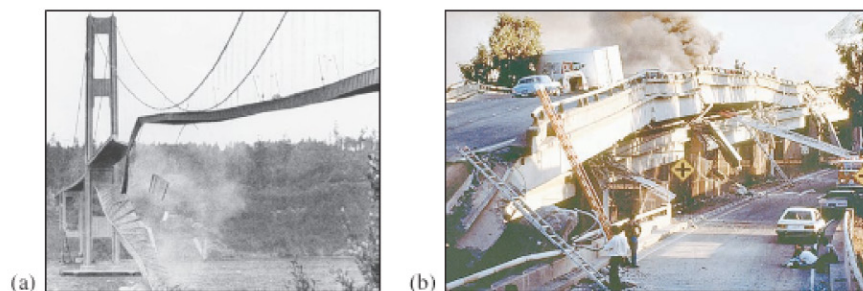**PHYSICS APPLIED**
Resonant collapse

FIGURE 11–19 (a) Large-amplitude oscillations of the Tacoma Narrows Bridge, due to gusty winds, led to its collapse (November 7, 1940). (b) Collapse of a freeway in California, due to the 1989 earthquake, in which resonance played a part.