If we substitute $k = mg/L$ into Eq. 11–7a, we obtain the period of a simple pendulum:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}}$$

or

$$T = 2\pi\sqrt{\frac{L}{g}}. \qquad [\theta \text{ small}] \quad \textbf{(11–11a)}$$

Period, simple pendulum

The frequency is $f = 1/T$, so

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}. \qquad [\theta \text{ small}] \quad \textbf{(11–11b)}$$

Frequency, simple pendulum

The mass m of the pendulum bob does not appear in these formulas for T and f. Thus we have the surprising result that the period and frequency of a simple pendulum do not depend on the mass of the pendulum bob. You may have noticed this if you pushed a small child and a large one on the same swing.

We also see from Eq. 11–11a that the period of a pendulum does not depend on the amplitude (like any SHM, Section 11–3), as long as the amplitude θ is small. Galileo is said to have first noted this fact while watching a swinging lamp in the cathedral at Pisa (Fig. 11–13). This discovery led to the invention of the pendulum clock, the first really precise timepiece, which became the standard for centuries.

PHYSICS APPLIED
Pendulum clock

Because a pendulum does not undergo *precisely* SHM, the period does depend slightly on the amplitude—the more so for large amplitudes. The accuracy of a pendulum clock would be affected, after many swings, by the decrease in amplitude due to friction. But the mainspring in a pendulum clock (or the falling weight in a grandfather clock) supplies energy to compensate for the friction and to maintain the amplitude constant, so that the timing remains precise.

EXAMPLE 11–9 **Measuring *g*.** A geologist uses a simple pendulum that has a length of 37.10 cm and a frequency of 0.8190 Hz at a particular location on the Earth. What is the acceleration of gravity at this location?

APPROACH We can use the length L and frequency f of the pendulum in Eq. 11–11b, which contains our unknown, g.

SOLUTION We solve Eq. 11–11b for g and obtain

$$g = (2\pi f)^2 L = \left(6.283 \times 0.8190\,\mathrm{s}^{-1}\right)^2(0.3710\,\mathrm{m}) = 9.824\,\mathrm{m/s^2}.$$

EXERCISE D (*a*) Estimate the length of the pendulum in a grandfather clock that ticks once per second. (*b*) What would be the period of a clock with a 1.0-m-long pendulum?

Equations 11–11 apply to a simple pendulum—a concentrated mass at the end of a string of negligible mass—but not to the oscillation of, say, a baseball bat suspended from one end.

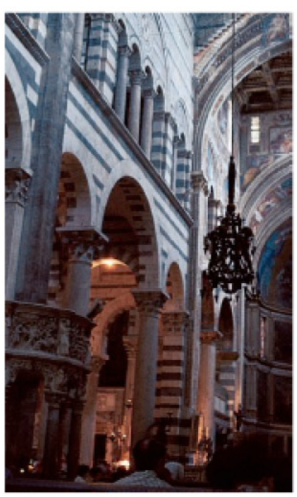

FIGURE 11–13 The swinging motion of this lamp, hanging by a very long cord from the ceiling of the cathedral at Pisa, is said to have been observed by Galileo and to have inspired him to the conclusion that the period of a pendulum does not depend on amplitude.