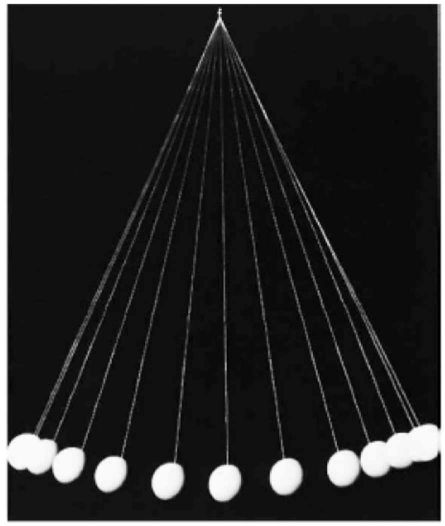

FIGURE 11–11 Strobe-light photo of an oscillating simple pendulum.

11–4 The Simple Pendulum

A **simple pendulum** consists of a small object (the pendulum bob) suspended from the end of a lightweight cord, Fig. 11–11. We assume that the cord doesn't stretch and that its mass can be ignored relative to that of the bob. The motion of a simple pendulum swinging back and forth with negligible friction resembles simple harmonic motion: the pendulum bob oscillates along the arc of a circle with equal amplitude on either side of its equilibrium point, and as it passes through the equilibrium point (where it would hang vertically) it has its maximum speed. But is it really undergoing SHM? That is, is the restoring force proportional to its displacement? Let us find out.

The displacement of the pendulum along the arc is given by $x = L\theta$, where θ is the angle the cord makes with the vertical and L is the length of the cord (Fig. 11–12). If the restoring force is proportional to x or to θ, the motion will be simple harmonic. The restoring force is the net force on the bob, equal to the component of the weight, mg, tangent to the arc:

$$F = -mg\sin\theta,$$

where g is the acceleration of gravity. The minus sign here, as in Eq. 11–1, means the force is in the direction opposite to the angular displacement θ. Since F is proportional to the sine of θ and not to θ itself, the motion is *not* SHM. However, if θ is small, then $\sin\theta$ is very nearly equal to θ when the latter is specified in radians. This can be seen by noting in Fig. 11–12 that the arc length $x\ (= L\theta)$ is nearly the same length as the chord $(= L\sin\theta)$ indicated by the horizontal straight dashed line, *if θ is small*. For angles less than 15°, the difference between θ (in radians) and $\sin\theta$ is less than 1%—see Table 11–1. Thus, to a very good approximation for small angles,

$$F = -mg\sin\theta \approx -mg\theta.$$

Substituting $x = L\theta$, or $\theta = x/L$, we have

$$F \approx -\frac{mg}{L}x.$$

Thus, for small displacements, the motion is essentially simple harmonic, since this equation fits Hooke's law, $F = -kx$. The effective force constant is $k = mg/L$.

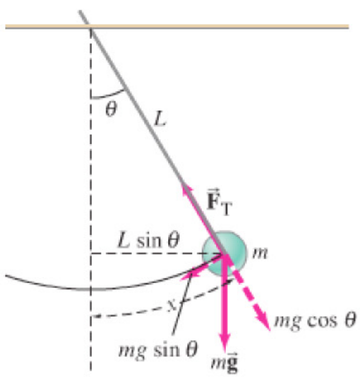

FIGURE 11–12 Simple pendulum, and a free-body diagram.

TABLE 11–1
Sin θ at Small Angles

θ (degrees)	θ (radians)	$\sin\theta$	% Difference
0	0	0	0
1°	0.01745	0.01745	0.005%
5°	0.08727	0.08716	0.1%
10°	0.17453	0.17365	0.5%
15°	0.26180	0.25882	1.1%
20°	0.34907	0.34202	2.0%
30°	0.52360	0.50000	4.7%