* Velocity and Acceleration as Functions of Time

Figure 11–10a, like Fig. 11–8, shows a graph of displacement x vs. time t, as given by Eqs. 11–8. We can also find the velocity v as a function of time from Fig. 11–6a. For the position shown (red dot in Fig. 11–6a), we see that the magnitude of v is $v_{\max}\sin\theta$, but $\vec{\mathbf{v}}$ points to the left, so $v = -v_{\max}\sin\theta$. Again setting $\theta = \omega t = 2\pi f t = 2\pi t/T$, we have

$$v = -v_{\max}\sin\omega t = -v_{\max}\sin(2\pi f t) = -v_{\max}\sin(2\pi t/T). \qquad \textbf{(11–9)}$$

Just after $t = 0$, the velocity is negative (points to the left) and remains so until $t = \frac{1}{2}T$ (corresponding to $\theta = 180° = \pi$ radians). After $t = \frac{1}{2}T$ until $t = T$ the velocity is positive. The velocity as a function of time (Eq. 11–9) is plotted in Fig. 11–10b. From Eqs. 11–6 and 11–7b,

$$v_{\max} = 2\pi A f = A\sqrt{\frac{k}{m}}.$$

For a given spring–mass system, the maximum speed $v_{\max}$ is higher if the amplitude is larger, and always occurs as the mass passes the equilibrium point.

The acceleration as a function of time is found from Newton's second law:

$$a = \frac{F}{m} = \frac{-kx}{m} = -\left(\frac{kA}{m}\right)\cos\omega t = -a_{\max}\cos(2\pi t/T) \qquad \textbf{(11–10)}$$

where the maximum acceleration is

$$a_{\max} = kA/m.$$

Equation 11–10 is plotted in Fig. 11–10c. Because the acceleration of a SHO is *not* constant, the equations for uniformly accelerated motion do *not* apply to SHM.

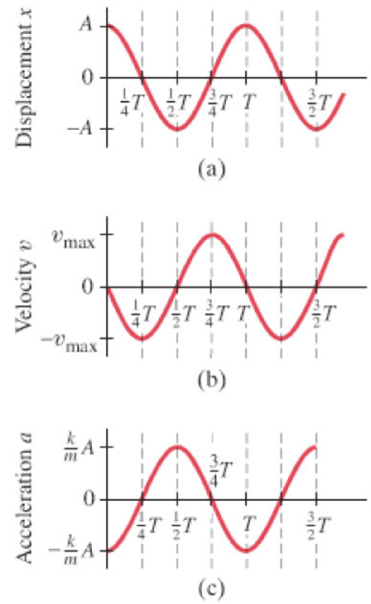

FIGURE 11–10 Graphs showing (a) displacement x as a function of time t: $x = A\cos(2\pi t/T)$; (b) velocity as a function of time: $v = -v_{\max}\sin(2\pi t/T)$; (c) acceleration as a function of time: $a = -(kA/m)\cos(2\pi t/T)$.

EXAMPLE 11–8 Loudspeaker. The cone of a loudspeaker vibrates in SHM at a frequency of 262 Hz ("middle C"). The amplitude at the center of the cone is $A = 1.5 \times 10^{-4}$ m, and at $t = 0$, $x = A$. (*a*) What equation describes the motion of the center of the cone? (*b*) What are the velocity and acceleration as a funcion of time? (*c*) What is the position of the cone at $t = 1.00$ ms $(= 1.00 \times 10^{-3}$ s$)$?

APPROACH The motion begins ($t = 0$) with the cone at its maximum displacement ($x = A$ at $t = 0$). So we use the cosine function, $x = A\cos\omega t$, to describe this SHM.

SOLUTION (*a*) Here

$$\omega = 2\pi f = (6.28\text{ rad})(262\text{ s}^{-1}) = 1650\text{ rad/s}.$$

The motion is described as

$$x = A\cos(2\pi f t) = (1.5 \times 10^{-4}\text{ m})\cos(1650t).$$

(*b*) The maximum velocity, from Eq. 11–6, is $v_{\max} = 2\pi A f = 2\pi(1.5 \times 10^{-4}\text{ m})(262\text{ s}^{-1}) = 0.25\text{ m/s}$. Then by Eq. 11–9,

$$v = -(0.25\text{ m/s})\sin(1650t).$$

From Eqs. 11–10 and 11–7b, the maximum acceleration is $a_{\max} = (k/m)A = (2\pi f)^2 A = 4\pi^2(262\text{ s}^{-1})^2(1.5 \times 10^{-4}\text{ m}) = 410\text{ m/s}^2$, which is more than 40 g's. So

$$a = -(410\text{ m/s}^2)\cos(1650t).$$

(*c*) At $t = 1.00 \times 10^{-3}$ s, Eq. 11–8a gives us

$$x = A\cos\omega t = (1.5 \times 10^{-4}\text{ m})\cos[(1650\text{ rad/s})(1.00 \times 10^{-3}\text{ s})]$$
$$= (1.5 \times 10^{-4}\text{ m})\cos(1.65\text{ rad}) = -1.2 \times 10^{-5}\text{ m}.$$

NOTE Be sure your calculator is set in RAD mode, not DEG mode, for these $\cos\omega t$ calculations.

CAUTION

Always be sure your calculator is in the correct mode for angles