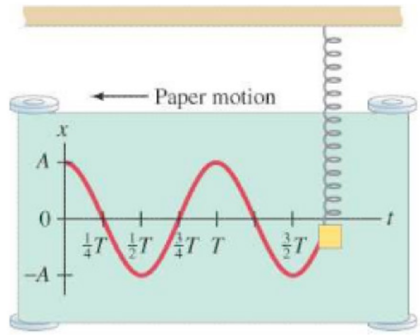
FIGURE 11–8 Position as a function of time $x = A\cos(2\pi t/T)$.



As we have seen, the x component of a uniformly rotating object's motion corresponds precisely to the motion of a simple harmonic oscillator. Thus Eqs. 11–8 give the position of an object undergoing simple harmonic motion. Since the cosine function varies between 1 and -1, x varies between A and $-A$, as it must. If a pen is attached to a vibrating mass as a sheet of paper is moved at a steady rate beneath it (Fig. 11–8), a curve will be drawn that accurately follows Eqs. 11–8.

EXAMPLE 11–7 **Starting with** $\boldsymbol{x = A\cos\omega t}$**.** The displacement of an object is described by the following equation, where x is in meters and t is in seconds:

$$x = (0.30\,\mathrm{m})\cos(8.0\,t).$$

Determine the oscillating object's (*a*) amplitude, (*b*) frequency, (*c*) period, (*d*) maximum speed, and (*e*) maximum acceleration.

APPROACH We start by comparing the given equation for x with Eq. 11–8b, $x = A\cos(2\pi f t)$.

SOLUTION From $x = A\cos(2\pi f t)$, we see by inspection that (*a*) the amplitude $A = 0.30\,\mathrm{m}$, and (*b*) $2\pi f = 8.0\,\mathrm{s}^{-1}$; so $f = (8.0\,\mathrm{s}^{-1}/2\pi) = 1.27\,\mathrm{Hz}$. (*c*) Then $T = 1/f = 0.79\,\mathrm{s}$. (*d*) The maximum speed (see Eq. 11–6) is

$$v_{\max} = 2\pi A f = (2\pi)(0.30\,\mathrm{m})(1.27\,\mathrm{s}^{-1}) = 2.4\,\mathrm{m/s}.$$

(*e*) The maximum acceleration, by Newton's second law, is $a_{\max} = F_{\max}/m = kA/m$, because $F\,(= kx)$ is greatest when x is greatest. From Eq. 11–7b we see that $k/m = (2\pi f)^2$. Hence

$$a_{\max} = \frac{k}{m}A = (2\pi f)^2 A = (2\pi)^2(1.27\,\mathrm{s}^{-1})^2(0.30\,\mathrm{m}) = 19\,\mathrm{m/s^2}.$$

Sinusoidal Motion

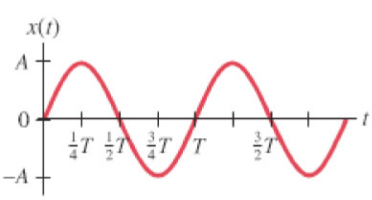


FIGURE 11–9 Sinusoidal nature of SHM as a function of time; in this case, $x = A\sin(2\pi t/T)$ because at $t = 0$ the mass is at the equilibrium position $x = 0$, but it also has (or is given) an initial speed at $t = 0$ that carries it to $x = A$ at $t = \frac{1}{4}T$.

SHM is sinusoidal

Equation 11–8, $x = A\cos\omega t$, assumes that the oscillating object starts from rest ($v = 0$) at its maximum displacement ($x = A$) at $t = 0$. Other equations for simple harmonic motion are also possible, depending on the initial conditions (when you choose t to be zero). For example, if at $t = 0$ the object is at the equilibrium position and the oscillations are begun by giving the object a push to the right ($+x$), the equation would be

$$x = A\sin\omega t = A\sin(2\pi t/T).$$

This curve (Fig. 11–9) has the same shape as the cosine curve shown in Fig. 11–8, except it is shifted to the right by a quarter cycle. Hence at $t = 0$ it starts out at $x = 0$ instead of at $x = A$.

Both sine and cosine curves are referred to as being **sinusoidal** (having the shape of a sine function). Thus simple harmonic motion† is said to be sinusoidal because the position varies as a sinusoidal function of time.

†Simple harmonic motion can be *defined* as motion that is sinusoidal. This definition is fully consistent with our earlier definition in Section 11–1.