Equation 11–7a is fully in accord with experiment and is valid not only for a spring, but for all kinds of simple harmonic motion—that is, for motion subject to a restoring force proportional to displacement, Eq. 11–1.

We can write the frequency using $f = 1/T$ (Eq. 11–2):

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}. \qquad \textbf{(11–7b)}$$

Frequency f of SHM

EXERCISE C Does a car bounce faster on its springs when empty or fully loaded?

EXAMPLE 11–6 ESTIMATE **Spider web.** A spider of mass 0.30 g waits in its web of negligible mass (Fig. 11–7). A slight movement causes the web to vibrate with a frequency of about 15 Hz. (*a*) Estimate the value of the spring stiffness constant k for the web. (*b*) At what frequency would you expect the web to vibrate if an insect of mass 0.10 g were trapped in addition to the spider?

APPROACH We can only make a rough estimate because a spider's web is fairly complicated and may vibrate with a mixture of frequencies. We use SHM as an approximate model.

SOLUTION (*a*) The frequency of SHM is given by Eq. 11–7b,

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

We solve for k:

$$\begin{aligned} k &= (2\pi f)^2 m \\ &= (6.28 \times 15\,\mathrm{s}^{-1})^2(3.0 \times 10^{-4}\,\mathrm{kg}) = 2.7\,\mathrm{N/m}. \end{aligned}$$

(*b*) The total mass is now $0.10\,\mathrm{g} + 0.30\,\mathrm{g} = 4.0 \times 10^{-4}\,\mathrm{kg}$. We could substitute $m = 4.0 \times 10^{-4}\,\mathrm{kg}$ into Eq. 11–7b. Instead, we notice that the frequency decreases with the square root of the mass. Since the new mass is 4/3 times the first mass, the frequency changes by a factor of $1/\sqrt{4/3} = \sqrt{3/4}$. Thus $f = (15\,\mathrm{Hz})(\sqrt{3/4}) = 13\,\mathrm{Hz}$.

NOTE Check this result by direct substitution of k, found in part (*a*), and the new mass m into Eq. 11–7b.

FIGURE 11–7 A spider waits for its prey (Example 11–6).

Position as a Function of Time

We now use the reference circle to find the position of a mass undergoing simple harmonic motion as a function of time. From Fig. 11–6, we see that $\cos\theta = x/A$, so the projection of the object's position on the x axis is

$$x = A\cos\theta.$$

Because the mass is rotating with angular velocity ω, we can write $\theta = \omega t$, where θ is in radians (Section 8–1). Thus

$$x = A\cos\omega t. \qquad \textbf{(11–8a)}$$

Position as a function of time (SHM)

Furthermore, since the angular velocity ω (specified in radians per second) can be written as $\omega = 2\pi f$, where f is the frequency (Eq. 8–7), we then write

$$x = A\cos(2\pi f t), \qquad \textbf{(11–8b)}$$

or in terms of the period T,

$$x = A\cos(2\pi t/T). \qquad \textbf{(11–8c)}$$

Notice in Eq. 11–8c that when $t = T$ (that is, after a time equal to one period), we have the cosine of 2π, which is the same as the cosine of zero. This makes sense since the motion repeats itself after a time $t = T$.

CAUTION
t is a variable (time);
T is a constant for a given situation