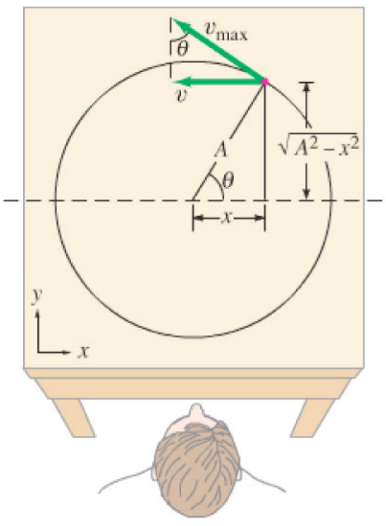

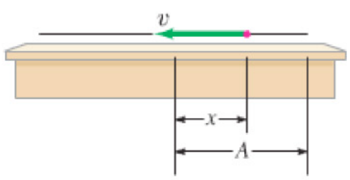

FIGURE 11–6 (a) Circular motion of a small (red) object. (b) Side view of circular motion (x component) is simple harmonic motion.

11–3 The Period and Sinusoidal Nature of SHM

The period of a simple harmonic oscillator is found to depend on the stiffness of the spring and also on the mass m that is oscillating. But—strange as it may seem—the *period does not depend on the amplitude*. You can find this out for yourself by using a watch and timing 10 or 20 cycles of an oscillating spring for a small amplitude and then for a large amplitude.

We can derive a formula for the period of simple harmonic motion (SHM) by comparing SHM to an object rotating in a circle. From this same "reference circle" we can obtain a second useful result—a formula for the position of an oscillating mass as a function of time. There is nothing actually rotating in a circle when a spring oscillates linearly, but it is the mathematical similarity that we find useful.

Period and Frequency

Consider a small object of mass m revolving counterclockwise in a circle of radius A, with constant speed v_{max}, on top of a table as shown in Fig. 11–6. As viewed from above, the motion is a circle in the xy plane. But a person who looks at the motion from the edge of the table sees an oscillatory motion back and forth, and this one-dimensional motion corresponds precisely to simple harmonic motion, as we shall now see.

What the person sees, and what we are interested in, is the projection of the circular motion onto the x axis (Fig. 11–6b). To see that this x-motion is analogous to SHM, let us calculate the magnitude of the x component of the velocity v_{max}, which is labeled v in Fig. 11–6. The two triangles involving θ in Fig. 11–6a are similar, so

$$\frac{v}{v_{max}} = \frac{\sqrt{A^2 - x^2}}{A}$$

or

$$v = v_{max}\sqrt{1 - \frac{x^2}{A^2}}.$$

This is exactly the equation for the speed of a mass oscillating with SHM, as we saw in Eq. 11–5. Thus the projection on the x axis of an object revolving in a circle has the same motion as a mass at the end of a spring.

We can now determine the period of SHM because it is equal to that of the revolving object making one complete revolution. First we note that the velocity v_{max} is equal to the circumference of the circle (distance) divided by the period T:

$$v_{max} = \frac{2\pi A}{T} = 2\pi A f. \qquad \textbf{(11–6)}$$

We solve for the period T:

$$T = \frac{2\pi A}{v_{max}}.$$

From energy conservation, Eqs. 11–4a and b, we have $\frac{1}{2}kA^2 = \frac{1}{2}mv^2_{max}$, so $A/v_{max} = \sqrt{m/k}$. Thus

Period T of SHM

$$T = 2\pi\sqrt{\frac{m}{k}}. \qquad \textbf{(11–7a)}$$

Period and frequency of SHM don't depend on amplitude

This is the formula we were looking for. The period depends on the mass m and the spring stiffness constant k, but not on the amplitude A. We see from Eq. 11–7a that the larger the mass, the longer the period; and the stiffer the spring (larger k), the shorter the period. This makes sense since a larger mass means more inertia and therefore slower response (smaller acceleration). And larger k means greater force and therefore quicker response (larger acceleration). Notice that Eq. 11–7a is not a direct proportion: the period varies as the *square root* of m/k. For example, the mass must be quadrupled to double the period.