EXAMPLE 11–4 **Spring calculations.** A spring stretches 0.150 m when a 0.300-kg mass is gently lowered on it as in Fig. 11–3b. The spring is then set up horizontally with the 0.300-kg mass resting on a frictionless table as in Fig. 11–5. The mass is pulled so that the spring is stretched 0.100 m from the equilibrium point, and released from rest. Determine (*a*) the spring stiffness constant k, (*b*) the amplitude of the horizontal oscillation A, (*c*) the magnitude of the maximum velocity v_{max}, (*d*) the magnitude of the velocity v when the mass is 0.050 m from equilibrium, and (*e*) the magnitude of the maximum acceleration a_{max} of the mass.

APPROACH When the 0.300-kg mass hangs at rest from the spring as in Fig. 11–3b, we apply Newton's second law for the vertical forces: $\Sigma F = 0 = mg - kx_0$, so $k = mg/x_0$. For the horizontal oscillations, the amplitude is given, the velocities are found using conservation of energy, and the acceleration from $F = ma$.

SOLUTION (*a*) The spring stretches 0.150 m due to the 0.300-kg load, so

$$k = \frac{F}{x_0} = \frac{mg}{x_0} = \frac{(0.300\,\mathrm{kg})(9.80\,\mathrm{m/s^2})}{0.150\,\mathrm{m}} = 19.6\,\mathrm{N/m}.$$

(*b*) The spring is now horizontal (on a table). It is stretched 0.100 m from equilibrium and is given no initial speed, so $A = 0.100\,\mathrm{m}$.

(*c*) The maximum velocity v_{max} is attained as the mass passes through the equilibrium point where all the energy is kinetic. By comparing the total energy (see Eq. 11–3) at equilibrium with that at full extension, conservation of energy tells us that

$$\tfrac{1}{2}mv_{max}^2 + 0 = 0 + \tfrac{1}{2}kA^2,$$

where $A = 0.100\,\mathrm{m}$. (Or, compare Eqs. 11–4a and b.) Solving for v_{max}, we have

$$v_{max} = A\sqrt{\frac{k}{m}} = (0.100\,\mathrm{m})\sqrt{\frac{19.6\,\mathrm{N/m}}{0.300\,\mathrm{kg}}} = 0.808\,\mathrm{m/s}.$$

(*d*) We use conservation of energy, or Eq. 11–5 derived from it, and find that

$$v = v_{max}\sqrt{1 - \frac{x^2}{A^2}} = (0.808\,\mathrm{m/s})\sqrt{1 - \frac{(0.050\,\mathrm{m})^2}{(0.100\,\mathrm{m})^2}} = 0.70\,\mathrm{m/s}.$$

(*e*) By Newton's second law, $F = ma$. So the maximum acceleration occurs where the force is greatest—that is, when $x = A = 0.100\,\mathrm{m}$. Thus

$$a_{max} = \frac{F_{max}}{m} = \frac{kA}{m} = \frac{(19.6\,\mathrm{N/m})(0.100\,\mathrm{m})}{0.300\,\mathrm{kg}} = 6.53\,\mathrm{m/s^2}.$$

NOTE We cannot use the kinematic equations, Eqs. 2–11, because the acceleration is not constant in SHM.

EXAMPLE 11–5 **More spring calculations—energy.** For the simple harmonic oscillator of Example 11–4, determine (*a*) the total energy, and (*b*) the kinetic and potential energies at half amplitude ($x = \pm A/2$).

APPROACH We use conservation of energy for a mass–spring system, Eqs. 11–3 and 11–4.

SOLUTION (*a*) With $k = 19.6\,\mathrm{N/m}$ and $A = 0.100\,\mathrm{m}$, the total energy E from Eq. 11–4a is

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(19.6\,\mathrm{N/m})(0.100\,\mathrm{m})^2 = 9.80 \times 10^{-2}\,\mathrm{J}.$$

(*b*) At $x = A/2 = 0.050\,\mathrm{m}$, we have

$$\mathrm{PE} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(19.6\,\mathrm{N/m})(0.050\,\mathrm{m})^2 = 2.5 \times 10^{-2}\,\mathrm{J}.$$

By conservation of energy, the kinetic energy must be

$$\mathrm{KE} = E - \mathrm{PE} = 7.3 \times 10^{-2}\,\mathrm{J}.$$