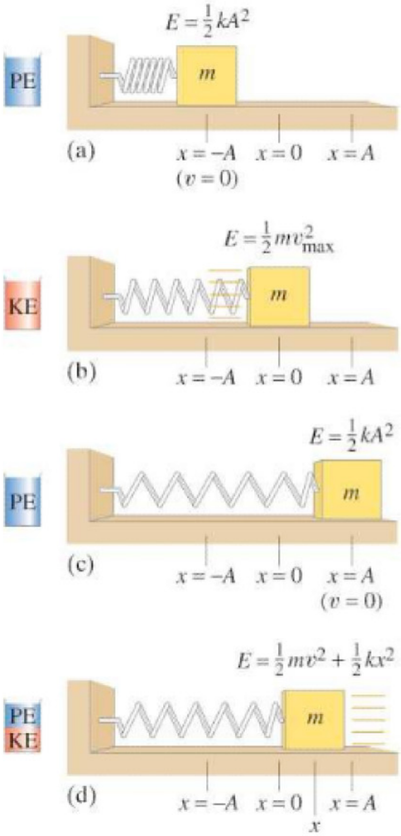

FIGURE 11–5 Energy changes from potential energy to kinetic energy and back again as the spring oscillates. Energy "buckets" (on the left) are described in Section 6–7.

remains constant. As the mass oscillates back and forth, the energy continuously changes from potential energy to kinetic energy, and back again (Fig. 11–5). At the extreme points, $x = -A$ and $x = A$ (Fig. 11–5a, c), all the energy is stored in the spring as potential energy (and is the same whether the spring is compressed or stretched to the full amplitude). At these extreme points, the mass stops momentarily as it changes direction, so $v = 0$ and

$$E = \tfrac{1}{2}m(0)^2 + \tfrac{1}{2}kA^2 = \tfrac{1}{2}kA^2. \tag{11–4a}$$

Thus, the **total mechanical energy of a simple harmonic oscillator is proportional to the square of the amplitude**. At the equilibrium point, $x = 0$ (Fig. 11–5b), all the energy is kinetic:

$$E = \tfrac{1}{2}mv_{\max}^2 + \tfrac{1}{2}k(0)^2 = \tfrac{1}{2}mv_{\max}^2, \tag{11–4b}$$

where $v_{\max}$ represents the *maximum* velocity during the motion (which occurs at $x = 0$). At intermediate points (Fig. 11–5d), the energy is part kinetic and part potential; because energy is conserved (we use Eqs. 11–3 and 11–4a),

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2. \tag{11–4c}$$

From this conservation of energy equation, we can obtain the velocity as a function of position. Solving for v^2, we have

$$v^2 = \frac{k}{m}\left(A^2 - x^2\right) = \frac{k}{m}A^2\left(1 - \frac{x^2}{A^2}\right).$$

From Eqs. 11–4a and 11–4b, we have $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kA^2$, so $v_{\max}^2 = (k/m)A^2$. Inserting this into the equation above and taking the square root, we have

$$v = \pm\, v_{\max}\sqrt{1 - \frac{x^2}{A^2}}. \tag{11–5}$$

This gives the velocity of the object at any position x. The object moves back and forth, so its velocity can be either in the + or − direction, but its magnitude depends only on the magnitude of x.

CONCEPTUAL EXAMPLE 11–3 **Doubling the amplitude.** Suppose the spring in Fig. 11–5 is stretched twice as far (to $x = 2A$). What happens to (*a*) the energy of the system, (*b*) the maximum velocity of the oscillating mass, (*c*) the maximum acceleration of the mass?

RESPONSE (*a*) From Eq. 11–4a, the total energy is proportional to the square of the amplitude A, so stretching it twice as far quadruples the energy $(2^2 = 4)$. You may protest, "I did work stretching the spring from $x = 0$ to $x = A$. Don't I do the same work stretching it from A to $2A$?" No. The force you exert is proportional to the displacement x, so for the second displacement, from $x = A$ to $2A$, you do more work than for the first displacement ($x = 0$ to A).
(*b*) From Eq. 11–4b, we can see that since the energy is quadrupled, the maximum velocity must be doubled. $\left[v_{\max} \propto \sqrt{E} \propto A.\right]$
(*c*) Since the force is twice as great when we stretch the spring twice as far, the acceleration is also twice as great: $a \propto F \propto x$.

EXERCISE B Suppose the spring in Fig. 11–5 is compressed to $x = -A$, but is given a push to the right so that the initial speed of the mass m is v_0. What effect does this push have on (*a*) the energy of the system, (*b*) the maximum velocity, (*c*) the maximum acceleration?