

**EXAMPLE 11-1** **Car springs.** When a family of four with a total mass of 200 kg step into their 1200-kg car, the car's springs compress 3.0 cm. (a) What is the spring constant of the car's springs (Fig. 11-4), assuming they act as a single spring? (b) How far will the car lower if loaded with 300 kg rather than 200 kg?

**APPROACH** We use Hooke's law. The extra force equal to the weight of the people,  $mg$ , causes a 3.0-cm displacement.

**SOLUTION** (a) The added force of  $(200 \text{ kg})(9.8 \text{ m/s}^2) = 1960 \text{ N}$  causes the springs to compress  $3.0 \times 10^{-2} \text{ m}$ . Therefore (Eq. 11-1), the spring constant is

$$k = \frac{F}{x} = \frac{1960 \text{ N}}{3.0 \times 10^{-2} \text{ m}} = 6.5 \times 10^4 \text{ N/m}.$$

(b) If the car is loaded with 300 kg, Hooke's law gives

$$x = \frac{F}{k} = \frac{(300 \text{ kg})(9.8 \text{ m/s}^2)}{(6.5 \times 10^4 \text{ N/m})} = 4.5 \times 10^{-2} \text{ m},$$

or 4.5 cm.

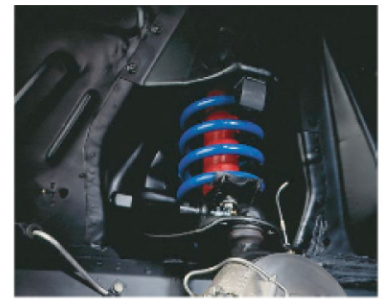
**NOTE** We could have obtained  $x$  without solving for  $k$ : since  $x$  is proportional to  $F$ , if 200 kg compresses the spring 3.0 cm, then 1.5 times the force will compress the spring 1.5 times as much, or 4.5 cm.

Any vibrating system for which the restoring force is directly proportional to the negative of the displacement (as in Eq. 11-1,  $F = -kx$ ) is said to exhibit **simple harmonic motion** (SHM).<sup>†</sup> Such a system is often called a **simple harmonic oscillator** (SHO). We saw in Section 9-5 that most solid materials stretch or compress according to Eq. 11-1 as long as the displacement is not too great. Because of this, many natural vibrations are simple harmonic, or sufficiently close to it that they can be treated using this SHM model.

### CONCEPTUAL EXAMPLE 11-2 Is the motion simple harmonic?

Which of the following represent a simple harmonic oscillator: (a)  $F = -0.5x^2$ , (b)  $F = -2.3y$ , (c)  $F = 8.6x$ , (d)  $F = -4\theta$ ?

**RESPONSE** Both (b) and (d) represent simple harmonic oscillators because they give the force as minus a constant times a displacement. The displacement need not be  $x$ , but the minus sign is required to restore the system to equilibrium, which is why (c) is not a SHO.



**FIGURE 11-4** Photo of a car's spring. (Also visible is the shock absorber, in red—see Section 11-5.)

SHM  
SHO

## 11-2 Energy in the Simple Harmonic Oscillator

With forces that are not constant, such as here with simple harmonic motion, it is often convenient and useful to use the energy approach, as we saw in Chapter 6.

To stretch or compress a spring, work has to be done. Hence potential energy is stored in a stretched or compressed spring. Indeed, we have already seen in Section 6-4 that elastic potential energy is given by

$$\text{PE} = \frac{1}{2}kx^2.$$

The total mechanical energy  $E$  of a mass-spring system is the sum of the kinetic and potential energies,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2, \quad (11-3) \quad \text{Total energy of SHO}$$

where  $v$  is the velocity of the mass  $m$  when it is a distance  $x$  from the equilibrium position. As long as there is no friction, the total mechanical energy  $E$

<sup>†</sup>The word "harmonic" refers to the motion being *sinusoidal*, which we discuss in Section 11-3. It is "simple" when there is sinusoidal motion of a single frequency.