

FIGURE 11-2 Force on, and velocity of, a mass at different positions of its oscillation cycle on a frictionless surface.



CAUTION

For vertical spring, measure displacement (x or y) from the vertical equilibrium position

Let us examine what happens when our uniform spring is initially compressed a distance $x = -A$, as shown in Fig. 11-2a, and then released. The spring exerts a force on the mass that pushes it toward the equilibrium position. But because the mass has been accelerated by the force, it passes the equilibrium position with considerable speed. Indeed, as the mass reaches the equilibrium position, the force on it decreases to zero, but its speed at this point is a maximum, v_{\max} , Fig. 11-2b. As the mass moves farther to the right, the force on it acts to slow it down, and it stops momentarily at $x = A$, Fig. 11-2c. It then begins moving back in the opposite direction, accelerating until it passes the equilibrium point, Fig. 11-2d, and then slows down until it reaches zero speed at the original starting point, $x = -A$, Fig. 11-2e. It then repeats the motion, moving back and forth symmetrically between $x = A$ and $x = -A$.

EXERCISE A An object is oscillating back and forth. Which of the following statements are true at some time during the course of the motion? (a) The object can have zero velocity and, simultaneously, nonzero acceleration. (b) The object can have zero velocity and, simultaneously, zero acceleration. (c) The object can have zero acceleration and, simultaneously, nonzero velocity. (d) The object can have nonzero velocity and nonzero acceleration simultaneously.

To discuss vibrational motion, we need to define a few terms. The distance x of the mass from the equilibrium point at any moment is called the **displacement**. The maximum displacement—the greatest distance from the equilibrium point—is called the **amplitude**, A . One **cycle** refers to the complete to-and-fro motion from some initial point back to that same point—say, from $x = -A$ to $x = A$ and back to $x = -A$. The **period**, T , is defined as the time required to complete one cycle. Finally, the **frequency**, f , is the number of complete cycles per second. Frequency is generally specified in hertz (Hz), where $1 \text{ Hz} = 1 \text{ cycle per second (s}^{-1}\text{)}$. It is easy to see, from their definitions, that frequency and period are inversely related, as we saw earlier (Eqs. 5-2 and 8-8):

$$f = \frac{1}{T} \quad \text{and} \quad T = \frac{1}{f}; \quad (11-2)$$

for example, if the frequency is 5 cycles per second, then each cycle takes $\frac{1}{5} \text{ s}$.

The oscillation of a spring hung vertically is essentially the same as that of a horizontal spring. Because of gravity, the length of a vertical spring with a mass m on the end will be longer at equilibrium than when that same spring is horizontal, as shown in Fig. 11-3. The spring is in equilibrium when $\Sigma F = 0 = mg - kx_0$, so the spring stretches an extra amount $x_0 = mg/k$ to be in equilibrium. If x is measured from this new equilibrium position, Eq. 11-1 can be used directly with the same value of k .

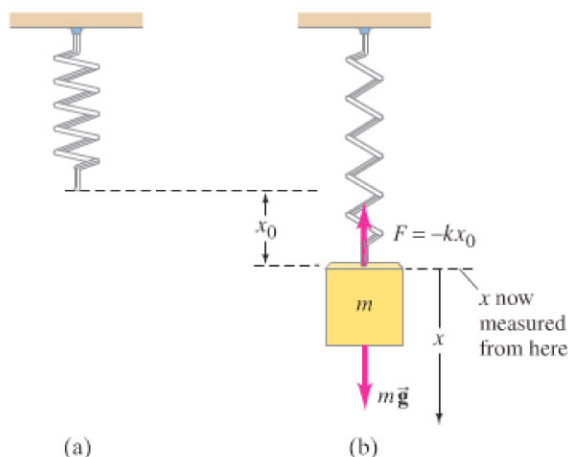


FIGURE 11-3

(a) Free spring, hung vertically.
(b) Mass m attached to spring in new equilibrium position, which occurs when $\Sigma F = 0 = mg - kx_0$.