11–1 Simple Harmonic Motion

When an object **vibrates** or **oscillates** back and forth, over the same path, each vibration taking the same amount of time, the motion is **periodic**. The simplest form of periodic motion is represented by an object oscillating on the end of a uniform coil spring. Because many other types of vibrational motion closely resemble this system, we will look at it in detail. We assume that the mass of the spring can be ignored, and that the spring is mounted horizontally, as shown in Fig. 11–1a, so that the object of mass m slides without friction on the horizontal surface. Any spring has a natural length at which it exerts no force on the mass m. The position of the mass at this point is called the **equilibrium position**. If the mass is moved either to the left, which compresses the spring, or to the right, which stretches it, the spring exerts a force on the mass that acts in the direction of returning the mass to the equilibrium position; hence it is called a *restoring force*. We consider the common situation where we can assume the magnitude of the restoring force F is directly proportional to the displacement x the spring has been stretched (Fig. 11–1b) or compressed (Fig. 11–1c) from the equilibrium position:

Equilibrium position

$$F = -kx. \qquad \text{[force exerted by spring]} \qquad \textbf{(11–1)}$$

Note that the equilibrium position has been chosen at $x = 0$. Equation 11–1, which is often referred to as Hooke's law (see Sections 6–4 and 9–5), is accurate as long as the spring is not compressed to the point where the coils are close to touching, or stretched beyond the elastic region (see Fig. 9–19).

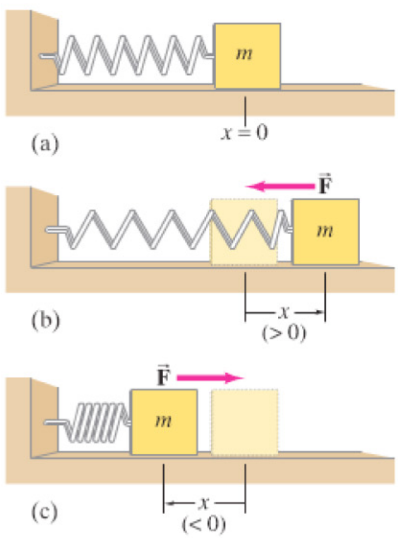

FIGURE 11–1 A mass vibrating at the end of a uniform spring.

The minus sign in Eq. 11–1 indicates that the restoring force is always in the direction opposite to the displacement x. For example, if we choose the positive direction to the right in Fig. 11–1, x is positive when the spring is stretched, but the direction of the restoring force is to the left (negative direction). If the spring is compressed, x is negative (to the left) but the force F acts toward the right (Fig. 11–1c).

The proportionality constant k in Eq. 11–1 is called the *spring constant* or *spring stiffness constant*. To stretch the spring a distance x, one has to exert an (external) force on the free end of the spring at least equal to

$$F = +kx. \qquad \text{[external force on spring]}$$

The greater the value of k, the greater the force needed to stretch a spring a given distance. That is, the stiffer the spring, the greater the spring constant k.

Note that the force F in Eq. 11–1 is *not* a constant, but varies with position. Therefore the acceleration of the mass m is not constant, so we *cannot* use the equations for constant acceleration developed in Chapter 2.

CAUTION

Force and acceleration are not *constant; Eqs. 2–11 are not useful here*