

Hence the proportionality constant for this equation is defined as the coefficient of viscosity,  $\eta$ :

$$F = \eta A \frac{v}{l}. \quad (10-8)$$

Solving for  $\eta$ , we find  $\eta = Fl/vA$ . The SI unit for  $\eta$  is  $\text{N} \cdot \text{s}/\text{m}^2 = \text{Pa} \cdot \text{s}$  (pascal·second). In the cgs system, the unit is  $\text{dyne} \cdot \text{s}/\text{cm}^2$ , which is called a *poise* (P). Viscosities are often given in centipoise ( $1 \text{ cP} = 10^{-2} \text{ P}$ ). Table 10-3 lists the coefficient of viscosity for various fluids. The temperature is also specified, since it has a strong effect; the viscosity of liquids such as motor oil, for example, decreases rapidly as temperature increases.<sup>‡</sup>

## \* 10-12 Flow in Tubes: Poiseuille's Equation, Blood Flow

If a fluid had no viscosity, it could flow through a level tube or pipe without a force being applied. Viscosity acts like a sort of friction, so a pressure difference between the ends of a level tube is necessary for the steady flow of any real fluid, be it water or oil in a pipe, or blood in the circulatory system of a human.

The rate of flow of a fluid in a round tube depends on the viscosity of the fluid, the pressure difference, and the dimensions of the tube. The French scientist J. L. Poiseuille (1799–1869), who was interested in the physics of blood circulation (and after whom the “poise” is named), determined how the variables affect the flow rate of an incompressible fluid undergoing laminar flow in a cylindrical tube. His result, known as *Poiseuille's equation*, is:

$$Q = \frac{\pi R^4 (P_1 - P_2)}{8\eta L}, \quad (10-9)$$

where  $R$  is the inside radius of the tube,  $L$  is its length,  $P_1 - P_2$  is the pressure difference between the ends,  $\eta$  is the coefficient of viscosity, and  $Q$  is the volume rate of flow (volume of fluid flowing past a given point per unit time which in SI has units of  $\text{m}^3/\text{s}$ ). Equation 10-9 applies only to laminar flow.

Poiseuille's equation tells us that the flow rate  $Q$  is directly proportional to the “pressure gradient,”  $(P_1 - P_2)/L$ , and it is inversely proportional to the viscosity of the fluid. This is just what we might expect. It may be surprising, however, that  $Q$  also depends on the *fourth* power of the tube's radius. This means that for the same pressure gradient, if the tube radius is halved, the flow rate is decreased by a factor of 16! Thus the rate of flow, or alternately the pressure required to maintain a given flow rate, is greatly affected by only a small change in tube radius.

An interesting example of this  $R^4$  dependence is *blood flow* in the human body. Poiseuille's equation is valid only for the streamline flow of an incompressible fluid with constant viscosity  $\eta$ . So it cannot be precisely accurate for blood whose flow is not without turbulence and that contains blood cells (whose diameter is almost equal to that of a capillary). Hence  $\eta$  depends to a certain extent on the blood flow speed  $v$ . Nonetheless, Poiseuille's equation does give a reasonable first approximation. The body controls the flow of blood by means of tiny bands of muscle surrounding the arteries. Contraction of these muscles reduces the diameter of an artery and, because of the  $R^4$  term in Eq. 10-9, the flow rate is greatly reduced for only a small change in radius. Very small actions by these muscles can thus control precisely the flow of blood to

**TABLE 10-3**  
Coefficients of Viscosity

Fluid (temperature in C°)	Coefficient of Viscosity, $\eta$ (Pa · s) <sup>†</sup>
Water (0°)	$1.8 \times 10^{-3}$
(20°)	$1.0 \times 10^{-3}$
(100°)	$0.3 \times 10^{-3}$
Whole blood (37°)	$\approx 4 \times 10^{-3}$
Blood plasma (37°)	$\approx 1.5 \times 10^{-3}$
Ethyl alcohol (20°)	$1.2 \times 10^{-3}$
Engine oil (30°) (SAE 10)	$200 \times 10^{-3}$
Glycerine (20°)	$1500 \times 10^{-3}$
Air (20°)	$0.018 \times 10^{-3}$
Hydrogen (0°)	$0.009 \times 10^{-3}$
Water vapor (100°)	$0.013 \times 10^{-3}$

<sup>†</sup>  $1 \text{ Pa} \cdot \text{s} = 10 \text{ P} = 1000 \text{ cP}$ .

*Poiseuille's equation  
for flow rate in a tube*



### PHYSICS APPLIED

*Medicine—  
blood flow  
and  
heart disease*

<sup>‡</sup>The Society of Automotive Engineers assigns numbers to represent the viscosity of oils: 30 weight (SAE 30) is more viscous than 10 weight. Multigrade oils, such as 20–50, are designed to maintain viscosity as temperature increases; 20–50 means the oil is 20 wt when cool but is like a 50-wt pure oil when it is hot (engine running temperature).