

FIGURE 10-24 Torricelli's theorem: $v_1 = \sqrt{2g(y_2 - y_1)}$.

Torricelli's theorem

10-10 Applications of Bernoulli's Principle: from Torricelli to Airplanes, Baseballs, and TIA

Bernoulli's equation can be applied to many situations. One example is to calculate the velocity, v_1 , of a liquid flowing out of a spigot at the bottom of a reservoir, Fig. 10-24. We choose point 2 in Eq. 10-5 to be the top surface of the liquid. Assuming the diameter of the reservoir is large compared to that of the spigot, v_2 will be almost zero. Points 1 (the spigot) and 2 (top surface) are open to the atmosphere, so the pressure at both points is equal to atmospheric pressure: $P_1 = P_2$. Then Bernoulli's equation becomes

$$\frac{1}{2}\rho v_1^2 + \rho g y_1 = \rho g y_2$$

or

$$v_1 = \sqrt{2g(y_2 - y_1)}. \quad (10-6)$$

This result is called **Torricelli's theorem**. Although it is seen to be a special case of Bernoulli's equation, it was discovered a century earlier by Evangelista Torricelli. Equation 10-6 tells us that the liquid leaves the spigot with the same speed that a freely falling object would attain if falling from the same height. This should not be too surprising since the derivation of Bernoulli's equation relies on the conservation of energy.

Another special case of Bernoulli's equation arises when a fluid is flowing horizontally with no appreciable change in height; that is, $y_1 = y_2$. Then Eq. 10-5 becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2, \quad (10-7)$$

which tells us quantitatively that the speed is high where the pressure is low, and vice versa. It explains many common phenomena, some of which are illustrated in Figs. 10-25 to 10-31. The pressure in the air blown at high speed across the top of the vertical tube of a perfume atomizer (Fig. 10-25a) is less than the normal air pressure acting on the surface of the liquid in the bowl. Thus atmospheric pressure in the bowl pushes the perfume up the tube because of the lower pressure at the top. A Ping-Pong ball can be made to float above a blowing jet of air (some vacuum cleaners can blow air), Fig. 10-25b; if the ball begins to leave the jet of air, the higher pressure in the still air outside the jet pushes the ball back in.

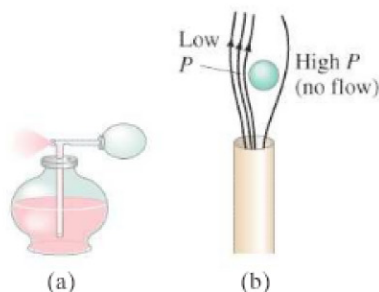
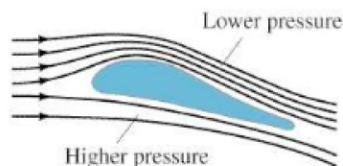


FIGURE 10-25 Examples of Bernoulli's principle: (a) atomizer, (b) Ping-Pong ball in jet of air.

FIGURE 10-26 Lift on an airplane wing. We are in the reference frame of the wing, seeing the air flow by.



PHYSICS APPLIED

Airplanes and dynamic lift

Airplane Wings and Dynamic Lift

Airplanes experience a "lift" force on their wings, keeping them up in the air, if they are moving at a sufficiently high speed relative to the air and the wing is tilted upward at a small angle (the "attack angle"), as in Fig. 10-26, where streamlines of air are shown rushing by the wing. (We are in the reference frame of the wing, as if sitting on the wing.) The upward tilt, as well as the rounded upper surface of the wing, causes the streamlines to be forced upward and to be crowded together above the wing. The area for air flow between any two streamlines is reduced as the streamlines get closer together, so from the equation of continuity ($A_1 v_1 = A_2 v_2$), the air speed increases above the wing where the streamlines are squished together. (Recall also how the crowded streamlines in a pipe constriction, Fig. 10-20, indicate the velocity is higher in the constriction.) Because the air speed is greater above the wing than below it, the pressure above the wing is less than the pressure below the wing (Bernoulli's principle). Hence there is a net upward force on the wing called **dynamic lift**. Experiments show that the speed of air above the wing can even be double the speed of the air below it. (Friction between the air and wing exerts a *drag force*, toward the rear, which must be overcome by the plane's engines.)