$$W_3 = -mg(y_2 - y_1),$$

where y_1 and y_2 are heights of the center of the tube above some (arbitrary) reference level. In the case shown in Fig. 10–23, this term is negative since the motion is uphill against the force of gravity. The net work W done on the fluid is thus

$$W = W_1 + W_2 + W_3$$
$$W = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mg y_2 + mg y_1.$$

According to the work-energy principle (Section 6–3), the net work done on a system is equal to its change in kinetic energy. Hence

$$\tfrac{1}{2} m v_2^2 - \tfrac{1}{2} m v_1^2 = P_1 A_1 \Delta l_1 - P_2 A_2 \Delta l_2 - mg y_2 + mg y_1.$$

The mass m has volume $A_1 \Delta l_1 = A_2 \Delta l_2$. Thus we can substitute $m = \rho A_1 \Delta l_1 = \rho A_2 \Delta l_2$, and then divide through by $A_1 \Delta l_1 = A_2 \Delta l_2$, to obtain

$$\tfrac{1}{2} \rho v_2^2 - \tfrac{1}{2} \rho v_1^2 = P_1 - P_2 - \rho g y_2 + \rho g y_1,$$

which we rearrange to get

$$P_1 + \tfrac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2} \rho v_2^2 + \rho g y_2. \qquad \textbf{(10–5)} \qquad \textit{Bernoulli's equation}$$

This is **Bernoulli's equation**. Since points 1 and 2 can be any two points along a tube of flow, Bernoulli's equation can be written as

$$P + \tfrac{1}{2} \rho v^2 + \rho g y = \text{constant}$$

at every point in the fluid, where y is the height of the center of the tube above a fixed reference level. [Note that if there is no flow ($v_1 = v_2 = 0$), then Eq. 10–5 reduces to the hydrostatic equation, Eq. 10–3b or c.]

Bernoulli's equation is an expression of the law of energy conservation, since we derived it from the work-energy principle.

EXERCISE C As water in a level pipe passes from a narrow cross section of pipe to a wider cross section, how does the pressure change?

EXAMPLE 10–13 **Flow and pressure in a hot-water heating system.** Water circulates throughout a house in a hot-water heating system. If the water is pumped at a speed of 0.50 m/s through a 4.0-cm-diameter pipe in the basement under a pressure of 3.0 atm, what will be the flow speed and pressure in a 2.6-cm-diameter pipe on the second floor 5.0 m above? Assume the pipes do not divide into branches.

APPROACH We use the equation of continuity at constant density to determine the flow speed on the second floor, and then Bernoulli's equation to find the pressure.

SOLUTION We take v_2 in the equation of continuity, Eq. 10–4, as the flow speed on the second floor, and v_1 as the flow speed in the basement. Noting that the areas are proportional to the radii squared ($A = \pi r^2$), we obtain

$$v_2 = \frac{v_1 A_1}{A_2} = \frac{v_1 \pi r_1^2}{\pi r_2^2} = (0.50 \text{ m/s}) \frac{(0.020 \text{ m})^2}{(0.013 \text{ m})^2} = 1.2 \text{ m/s}.$$

To find the pressure on the second floor, we use Bernoulli's equation:

$$P_2 = P_1 + \rho g (y_1 - y_2) + \tfrac{1}{2} \rho (v_1^2 - v_2^2)$$
$$= (3.0 \times 10^5 \text{ N/m}^2) + (1.0 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(-5.0 \text{ m})$$
$$\quad + \tfrac{1}{2}(1.0 \times 10^3 \text{ kg/m}^3)[(0.50 \text{ m/s})^2 - (1.2 \text{ m/s})^2]$$
$$= (3.0 \times 10^5 \text{ N/m}^2) - (4.9 \times 10^4 \text{ N/m}^2) - (6.0 \times 10^2 \text{ N/m}^2)$$
$$= 2.5 \times 10^5 \text{ N/m}^2 = 2.5 \text{ atm}.$$

NOTE The velocity term contributes very little in this case.