10–9 Bernoulli's Equation

Have you ever wondered why an airplane can fly, or how a sailboat can move against the wind? These are examples of a principle worked out by Daniel Bernoulli (1700–1782) concerning fluids in motion. In essence, **Bernoulli's principle** states that *where the velocity of a fluid is high, the pressure is low, and where the velocity is low, the pressure is high*. For example, if the pressures at points 1 and 2 in Fig. 10–20 are measured, it will be found that the pressure is lower at point 2, where the velocity is greater, than it is at point 1, where the velocity is smaller. At first glance, this might seem strange; you might expect that the greater speed at point 2 would imply a higher pressure. But this cannot be the case. For if the pressure at point 2 were higher than at 1, this higher pressure would slow the fluid down, whereas in fact it has sped up in going from point 1 to point 2. Thus the pressure at point 2 must be less than at point 1, to be consistent with the fact that the fluid accelerates. [To help clarify any misconceptions, a faster fluid *would* exert a greater force on an obstacle placed in its path if the fluid were stopped or bounced off the obstacle. But that is not what we mean by the pressure in a fluid, and besides we are not considering obstacles that interrupt the flow. We are examining smooth streamline flow. The fluid pressure is exerted in all directions, including on the walls of a pipe or surface of any material the fluid passes over.]

Bernoulli's principle

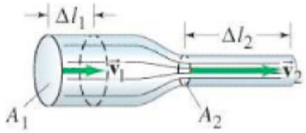

FIGURE 10–20 (repeated) Fluid flow through a pipe of varying diameter.

Bernoulli developed an equation that expresses this principle quantitatively. To derive Bernoulli's equation, we assume the flow is steady and laminar, the fluid is incompressible, and the viscosity is small enough to be ignored. To be general, we assume the fluid is flowing in a tube of nonuniform cross section that varies in height above some reference level, Fig. 10–23. We will consider the volume of fluid shown in color and calculate the work done to move it from the position shown in Fig. 10–23a to that shown in Fig. 10–23b. In this process, fluid at point 1 flows a distance Δl_1 and forces the fluid at point 2 to move a distance Δl_2. The fluid to the left of point 1 exerts a pressure P_1 on our section of fluid and does an amount of work

$$W_1 = F_1 \Delta l_1 = P_1 A_1 \Delta l_1 .$$

At point 2, the work done on our cross section of fluid is

$$W_2 = -P_2 A_2 \Delta l_2 .$$

The negative sign is present because the force exerted on the fluid is opposite to the motion (thus the fluid shown in color does work on the fluid to the right of point 2). Work is also done on the fluid by the force of gravity. The net effect of the process shown in Fig. 10–23 is to move a mass m of volume $A_1 \Delta l_1$ ($= A_2 \Delta l_2$, since the fluid is incompressible) from point 1 to point 2, so the work done by gravity is

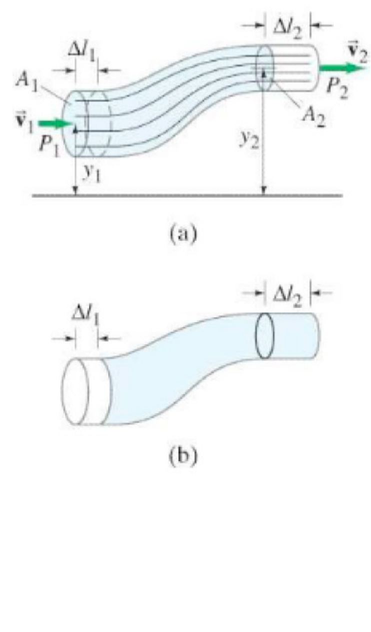

FIGURE 10–23 Fluid flow: for derivation of Bernoulli's equation.