If the fluid is incompressible ($\rho$ doesn't change with pressure), which is an excellent approximation for liquids under most circumstances (and sometimes for gases as well), then $\rho_1 = \rho_2$, and the equation of continuity becomes

$$A_1 v_1 = A_2 v_2. \qquad [\rho = \text{constant}] \quad \textbf{(10–4b)}$$

*Equation of continuity ($\rho$ = constant)*

The product $Av$ represents the *volume rate of flow* (volume of fluid passing a given point per second), since $\Delta V/\Delta t = A\,\Delta l/\Delta t = Av$, which in SI units is $m^3/s$. Equation 10–4b tells us that where the cross-sectional area is large, the velocity is small, and where the area is small, the velocity is large. That this is reasonable can be seen by looking at a river. A river flows slowly through a meadow where it is broad, but speeds up to torrential speed when passing through a narrow gorge.

**EXAMPLE 10–11 ESTIMATE Blood flow.** In humans, blood flows from the heart into the aorta, from which it passes into the major arteries. These branch into the small arteries (arterioles), which in turn branch into myriads of tiny capillaries, Fig. 10–21. The blood returns to the heart via the veins. The radius of the aorta is about 1.2 cm, and the blood passing through it has a speed of about 40 cm/s. A typical capillary has a radius of about $4 \times 10^{-4}$ cm, and blood flows through it at a speed of about $5 \times 10^{-4}$ m/s. Estimate the number of capillaries that are in the body.

PHYSICS APPLIED
*Blood flow*

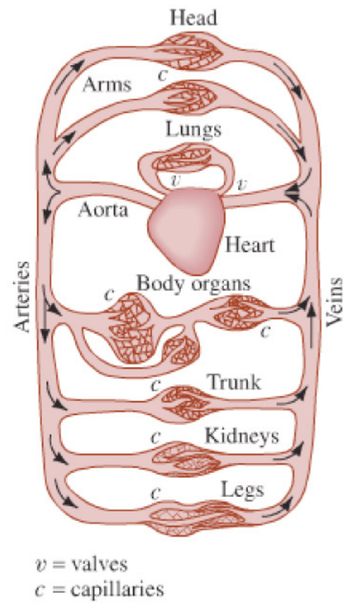



**FIGURE 10–21** Human circulatory system.

**APPROACH** We assume the density of blood doesn't vary significantly from the aorta to the capillaries. By the equation of continuity, the volume flow rate in the aorta must equal the volume flow rate through *all* the capillaries. The total area of all the capillaries is given by the area of one capillary multiplied by the total number $N$ of capillaries.

**SOLUTION** Let $A_1$ be the area of the aorta and $A_2$ be the area of *all* the capillaries through which blood flows. Then $A_2 = N\pi r_{\text{cap}}^2$, where $r_{\text{cap}} \approx 4 \times 10^{-4}$ cm is the estimated average radius of one capillary. From the equation of continuity (Eq. 10–4), we have

$$v_2 A_2 = v_1 A_1$$
$$v_2 N\pi r_{\text{cap}}^2 = v_1 \pi r_{\text{aorta}}^2$$

so

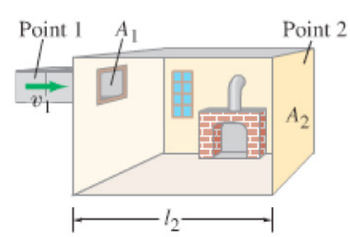
$$N = \frac{v_1}{v_2}\frac{r_{\text{aorta}}^2}{r_{\text{cap}}^2} = \left(\frac{0.40\text{ m/s}}{5 \times 10^{-4}\text{ m/s}}\right)\left(\frac{1.2 \times 10^{-2}\text{ m}}{4 \times 10^{-6}\text{ m}}\right)^2 \approx 7 \times 10^9,$$

or on the order of 10 billion capillaries.

**EXAMPLE 10–12 Heating duct to a room.** What area must a heating duct have if air moving 3.0 m/s along it can replenish the air every 15 minutes in a room of volume $300\text{ m}^3$? Assume the air's density remains constant.

PHYSICS APPLIED
*Heating duct*

**FIGURE 10–22** Example 10–12.




**APPROACH** We apply the equation of continuity at constant density, Eq. 10–4, to the air that flows through the duct (point 1 in Fig. 10–22) and then into the room (point 2). The volume flow rate in the room equals the volume of the room divided by the 15-minute replenishing time.

**SOLUTION** Consider the room as a large section of the duct, Fig. 10–22, and think of air equal to the volume of the room as passing by point 2 in $t$ = 15 minutes = 900 s. Reasoning in the same way we did to obtain Eq. 10–4a (changing $\Delta t$ to $t$), we write $v_2 = l_2/t$ so $A_2 v_2 = A_2 l_2/t = V_2/t$, where $V_2$ is the volume of the room. Then the equation of continuity becomes $A_1 v_1 = A_2 v_2 = V_2/t$ and

$$A_1 = \frac{V_2}{v_1 t} = \frac{300\text{ m}^3}{(3.0\text{ m/s})(900\text{ s})} = 0.11\text{ m}^2.$$

If the duct is square, then each side has length $l = \sqrt{A} = 0.33$ m, or 33 cm. A rectangular duct 20 cm × 55 cm will also do.