

10-8 Fluids in Motion; Flow Rate and the Equation of Continuity

We now turn to the subject of fluids in motion, which is called **fluid dynamics**, or (especially if the fluid is water) hydrodynamics. Many aspects of fluid motion are still being studied (for example, turbulence as a manifestation of chaos is a "hot" topic today). Nonetheless, with certain simplifying assumptions, we can understand a lot about this subject.

We can distinguish two main types of fluid flow. If the flow is smooth, such that neighboring layers of the fluid slide by each other smoothly, the flow is said to be streamline or laminar flow. In streamline flow, each particle of the fluid follows a smooth path, called a streamline, and these paths do not cross one another (Fig. 10–19a). Above a certain speed, the flow becomes turbulent. **Turbulent flow** is characterized by erratic, small, whirlpool-like circles called eddy currents or eddies (Fig. 10-19b). Eddies absorb a great deal of energy, and although a certain amount of internal friction called viscosity is present even during streamline flow, it is much greater when the flow is turbulent. A few tiny drops of ink or food coloring dropped into a moving liquid can quickly reveal whether the flow is streamline or turbulent.

Let us consider the steady laminar flow of a fluid through an enclosed tube or pipe as shown in Fig. 10-20. First we determine how the speed of the fluid changes when the size of the tube changes. The mass flow rate is defined as the mass Δm of fluid that passes a given point per unit time Δt :

mass flow rate =
$$\frac{\Delta m}{\Delta t}$$
.

In Fig. 10-20, the volume of fluid passing point 1 (that is, through area A_1) in a time Δt is $A_1 \Delta l_1$, where Δl_1 is the distance the fluid moves in time Δt . Since the velocity[‡] of fluid passing point 1 is $v_1 = \Delta l_1/\Delta t$, the mass flow rate $\Delta m_1/\Delta t$ through area A_1 is

$$\frac{\Delta m_1}{\Delta t} = \frac{\rho_1 \Delta V_1}{\Delta t} = \frac{\rho_1 A_1 \Delta l_1}{\Delta t} = \rho_1 A_1 v_1,$$

where $\Delta V_1 = A_1 \Delta l_1$ is the volume of mass Δm_1 , and ρ_1 is the fluid density. Similarly, at point 2 (through area A_2), the flow rate is $\rho_2 A_2 v_2$. Since no fluid flows in or out the sides, the flow rates through A_1 and A_2 must be equal. Thus, since

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$

then

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2. \tag{10-4a}$$

This is called the equation of continuity.

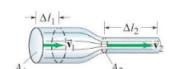


FIGURE 10-19

(b) turbulent flow.

FIGURE 10-20 Fluid flow through a pipe of varying diameter.

Equation of continuity (general)

[†]The word laminar means "in layers."

[‡]If there were no viscosity, the velocity would be the same across a cross section of the tube. Real fluids have viscosity, and this internal friction causes different layers of the fluid to flow at different speeds. In this case v_1 and v_2 represent the average speeds at each cross section.