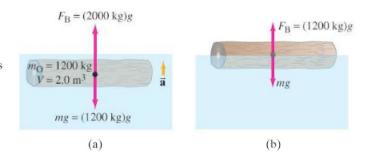
FIGURE 10–15 (a) The fully submerged log accelerates upward because $F_{\rm B} > mg$. It comes to equilibrium (b) when $\Sigma F = 0$, so $F_{\rm B} = mg = (1200~{\rm kg})g$. Thus $1200~{\rm kg}$, or $1.2~{\rm m}^3$, of water is displaced.



Floating

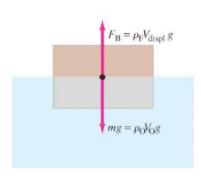
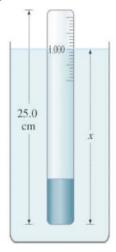


FIGURE 10–16 An object floating in equilibrium: $F_B = mg$.

Fraction of floating object submerged in water = its SG

FIGURE 10–17 A hydrometer. Example 10–9.



Archimedes' principle applies equally well to objects that float, such as wood. In general, an object floats on a fluid if its density is less than that of the fluid. This is readily seen from Fig. 10–15a, where a submerged object will experience a net upward force and float to the surface if $F_{\rm B} > mg$; that is, if $\rho_{\rm F} V g > \rho_{\rm O} V g$ or $\rho_{\rm F} > \rho_{\rm O}$. At equilibrium—that is, when floating—the buoyant force on an object has magnitude equal to the weight of the object. For example, a log whose specific gravity is 0.60 and whose volume is 2.0 m³ has a mass $m = \rho_{\rm O} V = (0.60 \times 10^3 \, {\rm kg/m^3})(2.0 \, {\rm m^3}) = 1200 \, {\rm kg}$. If the log is fully submerged, it will displace a mass of water $m_{\rm F} = \rho_{\rm F} V = (1000 \, {\rm kg/m^3})(2.0 \, {\rm m^3}) = 2000 \, {\rm kg}$. Hence the buoyant force on the log will be greater than its weight, and it will float upward to the surface (Fig. 10–15). The log will come to equilibrium when it displaces 1200 kg of water, which means that 1.2 m³ of its volume will be submerged. This 1.2 m³ corresponds to 60% of the volume of the log (1.2/2.0 = 0.60), so 60% of the log is submerged. In general when an object floats, we have $F_{\rm B} = mg$, which we can write as (see Fig. 10–16)

$$\rho_F V_{\text{displ}} g = \rho_O V_O g$$
,

where $V_{\rm O}$ is the full volume of the object and $V_{\rm displ}$ is the volume of fluid it displaces (= volume submerged). Thus

$$\frac{V_{\rm displ}}{V_{\rm O}} = \frac{\rho_{\rm O}}{\rho_{\rm F}} \cdot$$

That is, the fraction of the object submerged is given by the ratio of the object's density to that of the fluid. If the fluid is water, this fraction equals the specific gravity of the object.

EXAMPLE 10–9 Hydrometer calibration. A **hydrometer** is a simple instrument used to measure the specific gravity of a liquid by indicating how deeply the instrument sinks in the liquid. A particular hydrometer (Fig. 10–17) consists of a glass tube, weighted at the bottom, which is 25.0 cm long and 2.00 cm² in cross-sectional area, and has a mass of 45.0 g. How far from the end should the 1.000 mark be placed?

APPROACH The hydrometer will float in water if its density ρ is less than $\rho_{\rm w}=1.000~{\rm g/cm^3},~{\rm the~density~of~water}.$ The fraction of the hydrometer submerged $(V_{\rm displaced}/V_{\rm total})$ is equal to the density ratio $\rho/\rho_{\rm w}$.

SOLUTION The hydrometer has an overall density

$$\rho = \frac{m}{V} = \frac{45.0 \text{ g}}{(2.00 \text{ cm}^2)(25.0 \text{ cm})} = 0.900 \text{ g/cm}^3.$$

Thus, when placed in water, it will come to equilibrium when 0.900 of its volume is submerged. Since it is of uniform cross section, $(0.900)(25.0 \,\mathrm{cm}) = 22.5 \,\mathrm{cm}$ of its length will be submerged. The specific gravity of water is defined to be 1.000, so the mark should be placed $22.5 \,\mathrm{cm}$ from the end.