Archimedes is said to have discovered his principle in his bath while thinking how he might determine whether the king's new crown was pure gold or a fake. Gold has a specific gravity of 19.3, somewhat higher than that of most metals, but a determination of specific gravity or density is not readily done directly because, even if the mass is known, the volume of an irregularly shaped object is not easily calculated. However, if the object is weighed in air ($= w$) and also "weighed" while it is under water ($= w'$), the density can be determined using Archimedes' principle, as the following Example shows. The quantity w' is called the *apparent weight* in water, and is what a scale reads when the object is submerged in water (see Fig. 10–14); w' equals the true weight ($w = mg$) minus the buoyant force.

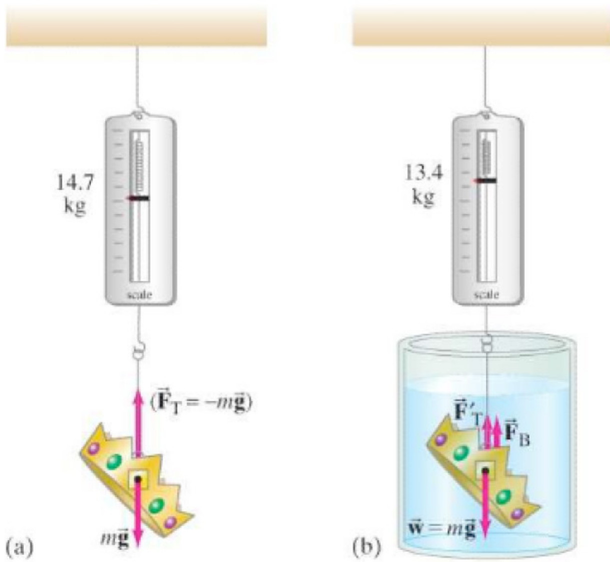

FIGURE 10–14 (a) A scale reads the mass of an object in air—in this case the crown of Example 10–8. All objects are at rest, so the tension F_T in the connecting cord equals the weight w of the object: $F_T = mg$. We show the free-body diagram of the crown, and F_T is what causes the scale reading (it's equal to the net downward force on the scale, by Newton's third law). (b) Submerged, the object has an additional force on it, the buoyant force F_B. The net force is zero, so $F'_T + F_B = mg\ (= w)$. The scale now reads $m' = 13.4$ kg, where m' is related to the effective weight by $w' = m'g$. Thus $F'_T = w' = w - F_B$.

EXAMPLE 10–8 **Archimedes: Is the crown gold?** When a crown of mass 14.7 kg is submerged in water, an accurate scale reads only 13.4 kg. Is the crown made of gold?

APPROACH If the crown is gold, its density and specific gravity must be very high, SG = 19.3 (see Section 10–2 and Table 10–1). We determine the specific gravity using Archimedes' principle and the two free-body diagrams shown in Fig. 10–14.

SOLUTION The apparent weight of the submerged object (the crown) is w', and it equals F'_T in Fig. 10–14b. The sum of the forces on the object is zero, so w' equals the actual weight $w\ (= mg)$ minus the buoyant force F_B:

$$w' = F'_T = w - F_B$$

so

$$w - w' = F_B.$$

Let V be the volume of the completely submerged object and ρ_O its density (so $\rho_O V$ is its mass), and let ρ_F be the density of the fluid (water). Then $(\rho_F V)g$ is the weight of fluid displaced ($= F_B$). Now we can write

Recall $m = \rho V$ (Eq. 10–1)

$$w = mg = \rho_O V g$$
$$w - w' = F_B = \rho_F V g.$$

We divide these two equations and obtain

$$\frac{w}{w - w'} = \frac{\rho_O V g}{\rho_F V g} = \frac{\rho_O}{\rho_F}.$$

We see that $w/(w - w')$ is equal to the specific gravity of the object if the fluid in which it is submerged is water ($\rho_F = 1.00 \times 10^3\ \mathrm{kg/m^3}$). Thus

$$\frac{\rho_O}{\rho_{H_2O}} = \frac{w}{w - w'} = \frac{(14.7\ \mathrm{kg})g}{(14.7\ \mathrm{kg} - 13.4\ \mathrm{kg})g} = \frac{14.7\ \mathrm{kg}}{1.3\ \mathrm{kg}} = 11.3.$$

This corresponds to a density of $11{,}300\ \mathrm{kg/m^3}$. The crown seems to be made of lead (see Table 10–1)!