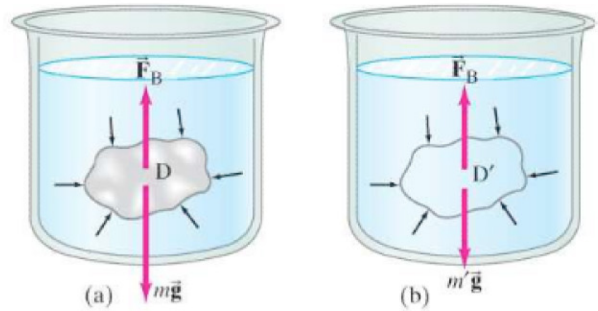
FIGURE 10–12 Archimedes' principle.



We can derive Archimedes' principle in general by the following simple but elegant argument. The irregularly shaped object D shown in Fig. 10–12a is acted on by the force of gravity (its weight, $m\vec{g}$, downward) and the buoyant force, $\vec{F}_B$, upward. We wish to determine F_B. To do so, we next consider a body (D′ in Fig. 10–12b), this time made of the fluid itself, with the same shape and size as the original object, and located at the same depth. You might think of this body of fluid as being separated from the rest of the fluid by an imaginary membrane. The buoyant force F_B on this body of fluid will be exactly the same as that on the original object since the surrounding fluid, which exerts F_B, is in exactly the same configuration. This body of fluid D′ is in equilibrium (the fluid as a whole is at rest). Therefore, $F_B = m'g$, where $m'g$ is the weight of the body of fluid. Hence the buoyant force F_B is equal to the weight of the body of fluid whose volume equals the volume of the original submerged object, which is Archimedes' principle.

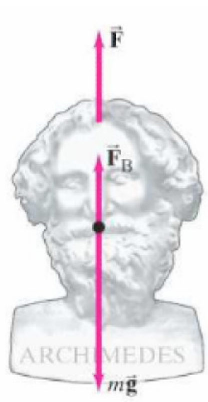
Archimedes' discovery was made by experiment. What we have done in the last two paragraphs is show that Archimedes' principle can be derived from Newtons' laws.

CONCEPTUAL EXAMPLE 10–6 **Two pails of water.** Consider two identical pails of water filled to the brim. One pail contains only water, the other has a piece of wood floating in it. Which pail has the greater weight?

RESPONSE Both pails weigh the same. Recall Archimedes' principle: the wood displaces a volume of water with weight equal to the weight of the wood. Some water will overflow the pail, but Archimedes' principle tells us the spilled water has weight equal to that of the wood; so the pails have the same weight.

EXAMPLE 10–7 **Recovering a submerged statue.** A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4\,\text{cm}^3$. How much force is needed to lift it?

FIGURE 10–13 Example 10–7. The force needed to lift the statue is $\vec{F}$.



APPROACH The force F needed to lift the statue is equal to the statue's weight mg minus the buoyant force F_B. Figure 10–13 is the free-body diagram.

SOLUTION The buoyant force on the statue due to the water is equal to the weight of $3.0 \times 10^4\,\text{cm}^3 = 3.0 \times 10^{-2}\,\text{m}^3$ of water (for seawater, $\rho = 1.025 \times 10^3\,\text{kg/m}^3$):

$$\begin{aligned} F_B = m_{H_2O}\,g &= \rho_{H_2O} V g \\ &= (1.025 \times 10^3\,\text{kg/m}^3)(3.0 \times 10^{-2}\,\text{m}^3)(9.8\,\text{m/s}^2) \\ &= 3.0 \times 10^2\,\text{N}. \end{aligned}$$

The weight of the statue is $mg = (70\,\text{kg})(9.8\,\text{m/s}^2) = 6.9 \times 10^2\,\text{N}$. Hence the force F needed to lift it is $690\,\text{N} - 300\,\text{N} = 390\,\text{N}$. It is as if the statue had a mass of only $(390\,\text{N})/(9.8\,\text{m/s}^2) = 40\,\text{kg}$.

NOTE Here $F = 390\,\text{N}$ is the force needed to lift the statue without acceleration when it is under water. As the statue comes *out* of the water, the force F increases, reaching 690 N when the statue is fully out of the water.