

FIGURE 10-3 Calculating the pressure at a depth h in a liquid.

Pressure variation with depth

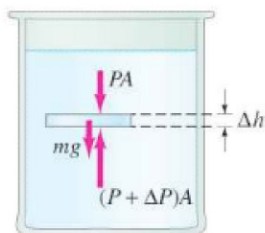


FIGURE 10-4 Forces on a thin slab of fluid (shown as a liquid, but it could instead be a gas).

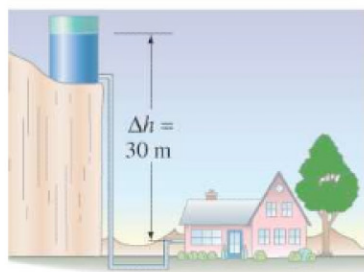
Change in pressure with change in depth in a fluid



PHYSICS APPLIED

Water supply

FIGURE 10-5 Example 10-3.



Let us now calculate quantitatively how the pressure in a liquid of uniform density varies with depth. Consider a point at a depth h below the surface of the liquid (that is, the surface is a height h above this point), as shown in Fig. 10-3. The pressure due to the liquid at this depth h is due to the weight of the column of liquid above it. Thus the force due to the weight of liquid acting on the area A is $F = mg = (\rho V)g = \rho Ahg$, where Ah is the volume of the column of liquid, ρ is the density of the liquid (assumed to be constant), and g is the acceleration of gravity. The pressure P due to the weight of liquid is then

$$P = \frac{F}{A} = \frac{\rho Ahg}{A}$$

$$P = \rho gh.$$

$$[\text{liquid}] \quad (10-3a)$$

Note that the area A doesn't affect the pressure at a given depth. The fluid pressure is directly proportional to the density of the liquid and to the depth within the liquid. In general, the pressure at equal depths within a uniform liquid is the same.

Equation 10-3a is extremely useful. It is valid for fluids whose density is constant and does not change with depth—that is, if the fluid is *incompressible*. This is usually a good approximation for liquids (although at great depths in the ocean, the density of water is increased substantially by compression due to the great weight of water above).

Gases, on the other hand, are very compressible, and density can vary significantly with depth. For this more general case, in which ρ may vary, Eq. 10-3a may not be useful. So let us consider a thin slab of liquid of volume $V = A \Delta h$ as shown in Fig. 10-4. We choose Δh thin enough so that ρ doesn't vary significantly over the small thickness Δh . Let P be the pressure exerted downward on the top surface, and let $P + \Delta P$ be the pressure upward on the bottom surface. The forces acting on our thin slab of fluid, as shown in Fig. 10-4, are $(P + \Delta P)A$ upward and PA downward, and the downward weight of the slab, $mg = (\rho V)g = \rho A \Delta h g$. We assume the fluid is at rest, so the net force on the slab is zero. Then

$$(P + \Delta P)A - PA - \rho A \Delta h g = 0.$$

The area A cancels from each term, and when we solve for ΔP we obtain

$$\Delta P = \rho g \Delta h. \quad [\rho \approx \text{constant over } \Delta h] \quad (10-3b)$$

Equation 10-3b tells us how the pressure changes over a small change in depth (Δh) within a fluid, even if compressible.

EXAMPLE 10-3 Pressure at a faucet. The surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house, Fig. 10-5. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.

APPROACH Water is practically incompressible, so ρ is constant even for a $\Delta h = 30$ m when used in Eq. 10-3b. Only Δh matters; we can ignore the "route" of the pipe and its bends.

SOLUTION The same atmospheric pressure acts both at the surface of the water in the storage tank and on the water leaving the faucet. So, the water pressure difference between the faucet and the surface of the water in the tank is

$$\begin{aligned} \Delta P &= \rho g \Delta h = (1.0 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(30 \text{ m}) \\ &= 2.9 \times 10^5 \text{ N/m}^2. \end{aligned}$$

NOTE The height h is sometimes called the **pressure head**. In this Example, the head of water is 30 m at the faucet. The very different diameters of the tank and faucet don't affect the result—only pressure does.

EXERCISE A A dam holds back a lake that is 85 m deep at the dam. If the lake is 20 km long, how much thicker should the dam be than if the lake were smaller, only 1.0 km long?