

nience, we set the positive direction of the x axis to be the direction of the throw (Fig. 9.2). Let us also assume that the x axis is tangent to the circular path of the spacecraft.

We take the system to consist of the astronaut and the uniform. Because of the gravitational force (which keeps the astronaut, his uniform, and the entire spacecraft in orbit), the system is not really isolated. However, this force is directed perpendicular to the motion of the system. Therefore, momentum is constant in the x direction because there are no external forces in this direction.

The total momentum of the system before the throw is zero ($m_1\mathbf{v}_{1i} + m_2\mathbf{v}_{2i} = 0$). Therefore, the total momentum after the throw must be zero; that is,

$$m_1\mathbf{v}_{1f} + m_2\mathbf{v}_{2f} = 0$$

With $m_1 = 70$ kg, $\mathbf{v}_{2f} = 20\mathbf{i}$ m/s, and $m_2 = 1$ kg, solving for \mathbf{v}_{1f} , we find the recoil velocity of the astronaut to be

$$\mathbf{v}_{1f} = -\frac{m_2}{m_1}\mathbf{v}_{2f} = -\left(\frac{1 \text{ kg}}{70 \text{ kg}}\right)(20\mathbf{i} \text{ m/s}) = -0.3\mathbf{i} \text{ m/s}$$

The negative sign for \mathbf{v}_{1f} indicates that the astronaut is moving to the left after the throw, in the direction opposite the direction of motion of the uniform, in accordance with Newton's third law. Because the astronaut is much more massive than his uniform, his acceleration and consequent velocity are much smaller than the acceleration and velocity of the uniform.

EXAMPLE 9.2 Breakup of a Kaon at Rest

One type of nuclear particle, called the *neutral kaon* (K^0), breaks up into a pair of other particles called *pions* (π^+ and π^-) that are oppositely charged but equal in mass, as illustrated in Figure 9.3. Assuming the kaon is initially at rest, prove that the two pions must have momenta that are equal in magnitude and opposite in direction.

Solution The breakup of the kaon can be written

$$K^0 \longrightarrow \pi^+ + \pi^-$$

If we let \mathbf{p}^+ be the momentum of the positive pion and \mathbf{p}^- the momentum of the negative pion, the final momentum of the system consisting of the two pions can be written

$$\mathbf{p}_f = \mathbf{p}^+ + \mathbf{p}^-$$

Because the kaon is at rest before the breakup, we know that $\mathbf{p}_i = 0$. Because momentum is conserved, $\mathbf{p}_i = \mathbf{p}_f = 0$, so that $\mathbf{p}^+ + \mathbf{p}^- = 0$, or

$$\mathbf{p}^+ = -\mathbf{p}^-$$

The important point behind this problem is that even though it deals with objects that are very different from those in the preceding example, the physics is identical: Linear momentum is conserved in an isolated system.

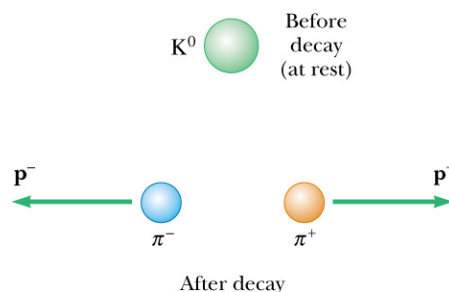


Figure 9.3 A kaon at rest breaks up spontaneously into a pair of oppositely charged pions. The pions move apart with momenta that are equal in magnitude but opposite in direction.

9.2 IMPULSE AND MOMENTUM

As we have seen, the momentum of a particle changes if a net force acts on the particle. Knowing the change in momentum caused by a force is useful in solving some types of problems. To begin building a better understanding of this important concept, let us assume that a single force \mathbf{F} acts on a particle and that this force may vary with time. According to Newton's second law, $\mathbf{F} = d\mathbf{p}/dt$, or

$$d\mathbf{p} = \mathbf{F} dt \quad (9.7)$$

We can integrate² this expression to find the change in the momentum of a particle when the force acts over some time interval. If the momentum of the particle

²Note that here we are integrating force with respect to time. Compare this with our efforts in Chapter 7, where we integrated force with respect to position to express the work done by the force.