

Because the time derivative of the total momentum $\mathbf{p}_{\text{tot}} = \mathbf{p}_1 + \mathbf{p}_2$ is *zero*, we conclude that the *total* momentum of the system must remain constant:

$$\mathbf{p}_{\text{tot}} = \sum_{\text{system}} \mathbf{p} = \mathbf{p}_1 + \mathbf{p}_2 = \text{constant} \quad (9.4)$$

or, equivalently,

$$\mathbf{p}_{1i} + \mathbf{p}_{2i} = \mathbf{p}_{1f} + \mathbf{p}_{2f} \quad (9.5)$$

where \mathbf{p}_{1i} and \mathbf{p}_{2i} are the initial values and \mathbf{p}_{1f} and \mathbf{p}_{2f} the final values of the momentum during the time interval dt over which the reaction pair interacts. Equation 9.5 in component form demonstrates that the total momenta in the x , y , and z directions are all independently conserved:

$$\sum_{\text{system}} p_{ix} = \sum_{\text{system}} p_{fx}} \quad \sum_{\text{system}} p_{iy} = \sum_{\text{system}} p_{fy} \quad \sum_{\text{system}} p_{iz} = \sum_{\text{system}} p_{fz} \quad (9.6)$$

This result, known as the **law of conservation of linear momentum**, can be extended to any number of particles in an isolated system. It is considered one of the most important laws of mechanics. We can state it as follows:

Conservation of momentum

Whenever two or more particles in an isolated system interact, the total momentum of the system remains constant.

This law tells us that **the total momentum of an isolated system at all times equals its initial momentum**.

Notice that we have made no statement concerning the nature of the forces acting on the particles of the system. The only requirement is that the forces must be *internal* to the system.

Quick Quiz 9.2

Your physical education teacher throws a baseball to you at a certain speed, and you catch it. The teacher is next going to throw you a medicine ball whose mass is ten times the mass of the baseball. You are given the following choices: You can have the medicine ball thrown with (a) the same speed as the baseball, (b) the same momentum, or (c) the same kinetic energy. Rank these choices from easiest to hardest to catch.

EXAMPLE 9.1 The Floating Astronaut

A SkyLab astronaut discovered that while concentrating on writing some notes, he had gradually floated to the middle of an open area in the spacecraft. Not wanting to wait until he floated to the opposite side, he asked his colleagues for a push. Laughing at his predicament, they decided not to help, and so he had to take off his uniform and throw it in one direction so that he would be propelled in the opposite direction. Estimate his resulting velocity.

Solution We begin by making some reasonable guesses of relevant data. Let us assume we have a 70-kg astronaut who threw his 1-kg uniform at a speed of 20 m/s. For convenience,



Figure 9.2 A hapless astronaut has discarded his uniform to get somewhere.