quantity of motion; this is perhaps a more graphic description than our present-day word *momentum,* which comes from the Latin word for movement.

Quick Quiz 9.1

Two objects have equal kinetic energies. How do the magnitudes of their momenta compare? (a) $p_1 < p_2$, (b) $p_1 = p_2$, (c) $p_1 > p_2$, (d) not enough information to tell.

Newton's second law for a particle

Using Newton's second law of motion, we can relate the linear momentum of a particle to the resultant force acting on the particle: **The time rate of change of the linear momentum of a particle is equal to the net force acting on the particle:**

$$\sum \mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} \qquad (9.3)$$

In addition to situations in which the velocity vector varies with time, we can use Equation 9.3 to study phenomena in which the mass changes. The real value of Equation 9.3 as a tool for analysis, however, stems from the fact that when the net force acting on a particle is zero, the time derivative of the momentum of the particle is zero, and therefore its linear momentum[1] is constant. Of course, if the particle is *isolated,* then by necessity $\Sigma \mathbf{F} = 0$ and $\mathbf{p}$ remains unchanged. This means that $\mathbf{p}$ is conserved. Just as the law of conservation of energy is useful in solving complex motion problems, the law of conservation of momentum can greatly simplify the analysis of other types of complicated motion.

Conservation of Momentum for a Two-Particle System

6.2 Consider two particles 1 and 2 that can interact with each other but are isolated from their surroundings (Fig. 9.1). That is, the particles may exert a force on each other, but no external forces are present. It is important to note the impact of Newton's third law on this analysis. If an internal force from particle 1 (for example, a gravitational force) acts on particle 2, then there must be a second internal force—equal in magnitude but opposite in direction—that particle 2 exerts on particle 1.

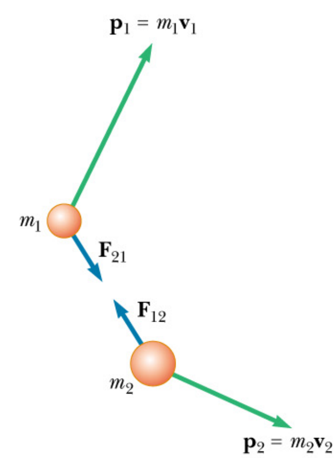

Figure 9.1 At some instant, the momentum of particle 1 is $\mathbf{p}_1 = m_1\mathbf{v}_1$ and the momentum of particle 2 is $\mathbf{p}_2 = m_2\mathbf{v}_2$. Note that $\mathbf{F}_{12} = -\mathbf{F}_{21}$. The total momentum of the system $\mathbf{p}_{\text{tot}}$ is equal to the vector sum $\mathbf{p}_1 + \mathbf{p}_2$.

Suppose that at some instant, the momentum of particle 1 is $\mathbf{p}_1$ and that of particle 2 is $\mathbf{p}_2$. Applying Newton's second law to each particle, we can write

$$\mathbf{F}_{21} = \frac{d\mathbf{p}_1}{dt} \qquad \text{and} \qquad \mathbf{F}_{12} = \frac{d\mathbf{p}_2}{dt}$$

where $\mathbf{F}_{21}$ is the force exerted by particle 2 on particle 1 and $\mathbf{F}_{12}$ is the force exerted by particle 1 on particle 2. Newton's third law tells us that $\mathbf{F}_{12}$ and $\mathbf{F}_{21}$ are equal in magnitude and opposite in direction. That is, they form an action–reaction pair $\mathbf{F}_{12} = -\mathbf{F}_{21}$. We can express this condition as

$$\mathbf{F}_{21} + \mathbf{F}_{12} = 0$$

or as

$$\frac{d\mathbf{p}_1}{dt} + \frac{d\mathbf{p}_2}{dt} = \frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2) = 0$$

[1]In this chapter, the terms *momentum* and *linear momentum* have the same meaning. Later, in Chapter 11, we shall use the term *angular momentum* when dealing with rotational motion.