FIGURE 9–29 Stones in a round (or "true") arch are mainly under compression.

FIGURE 9–30 Flying buttresses (on the cathedral of Notre Dame, in Paris).

The advantage of the "true" or round (semicircular) arch is that, if well designed, its wedge-shaped stones experience stress which is mainly compressive (Fig. 9–29) even when supporting a large load such as the wall and roof of a cathedral. A round arch consisting of many well-shaped stones could span a very wide space. However, considerable buttressing on the sides was needed to support the horizontal components of the forces, which we discuss shortly.

The pointed arch came into use about A.D. 1100 and became the hallmark of the great Gothic cathedrals. It too was an important technical innovation, and was first used to support heavy loads such as the tower of a cathedral, and as the central arch. Because of the steepness of the pointed arch, the forces due to the weight above could be brought down more nearly vertically, so less horizontal buttressing would be needed. The pointed arch reduced the load on the walls, so there could be more openness and light. The smaller buttressing needed was provided on the outside by graceful flying buttresses (Fig. 9–30).

The technical innovation of the pointed arch was achieved not through calculation but through experience and intuition; it was not until much later that detailed calculations, such as those presented earlier in this Chapter, came into use. To make an accurate analysis of a stone arch is quite difficult in practice. But if we make some simplifying assumptions, we can show why the horizontal component of the force at the base is less for a pointed arch than for a round one. Figure 9–31 shows a round arch and a pointed arch, each with an 8.0-m span. The height of the round arch is thus 4.0 m, whereas that of the pointed arch is larger and has been chosen to be 8.0 m. Each arch supports a weight of 12.0×10^4 N ($\approx 12,000$ kg $\times$ g), which, for simplicity, we have divided into two parts (each 6.0×10^4 N) acting on the two halves of each arch as shown. For the arch to be in equilibrium, each of the supports must exert an upward force of 6.0×10^4 N. Each support also exerts a horizontal force, F_H, at the base of the arch, and it is this we want to calculate. We focus only on the right half of each arch. We set equal to zero the total torque calculated about the apex of the arch due to the forces exerted on that half arch, as if there were a hinge at the apex. For the round arch, the torque equation ($\Sigma\tau = 0$) is

$$(4.0\,\text{m})(6.0 \times 10^4\,\text{N}) - (2.0\,\text{m})(6.0 \times 10^4\,\text{N}) - (4.0\,\text{m})(F_H) = 0.$$

Thus $F_H = 3.0 \times 10^4$ N for the round arch. For the pointed arch, the torque equation is

$$(4.0\,\text{m})(6.0 \times 10^4\,\text{N}) - (2.0\,\text{m})(6.0 \times 10^4\,\text{N}) - (8.0\,\text{m})(F_H) = 0.$$

Solving, we find that $F_H = 1.5 \times 10^4$ N—only half as much as for the round arch! From this calculation we can see that the horizontal buttressing force required for a pointed arch is less because the arch is higher, and there is therefore a longer lever arm for this force. Indeed, the steeper the arch, the less the horizontal component of the force needs to be, and hence the more nearly vertical is the force exerted at the base of the arch.
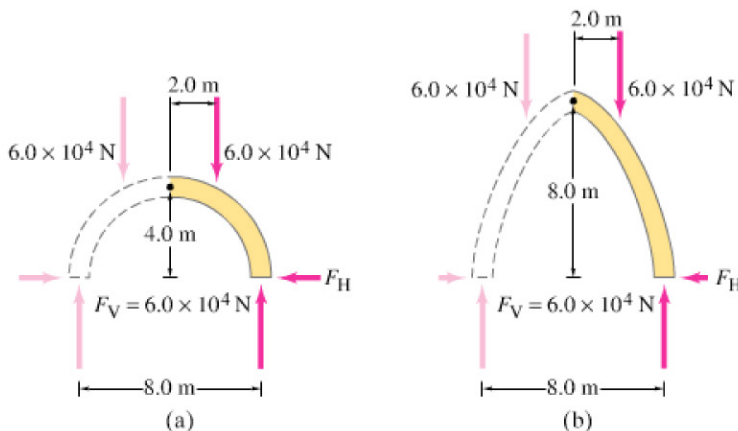
FIGURE 9–31 Forces in (a) a round arch, compared with those in (b) a pointed arch.