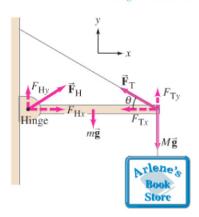**FIGURE 9–10** Example 9–6.

Our next Example involves a beam that is attached to a wall by a hinge and is supported by a cable or cord (Fig. 9–10). It is important to remember that a flexible cable can support a force only along its length. (If there were a component of force perpendicular to the cable, it would bend because it is flexible.) But for a rigid device, such as the hinge in Fig. 9–10, the force can be in any direction and we can know the direction only after solving the problem. The hinge is assumed small and smooth, so it can exert no internal torque (about its center) on the beam.

EXAMPLE 9–6 **Hinged beam and cable.** A uniform beam, 2.20 m long with mass $m = 25.0$ kg, is mounted by a hinge on a wall as shown in Fig. 9–10. The beam is held in a horizontal position by a cable that makes an angle $\theta = 30.0°$ as shown. The beam supports a sign of mass $M = 28.0$ kg suspended from its end. Determine the components of the force $\vec{F}_H$ that the hinge exerts on the beam, and the tension F_T in the supporting cable.

APPROACH Figure 9–10 is the free-body diagram for the beam, showing all the forces acting on the beam. It also shows the components of $\vec{F}_H$ and $\vec{F}_T$. We have three unknowns, F_{Hx}, F_{Hy}, and F_T (we are given θ), so we will need all three equations, $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma \tau = 0$.

SOLUTION The sum of the forces in the vertical (y) direction is

$$\Sigma F_y = 0$$
$$F_{Hy} + F_{Ty} - mg - Mg = 0. \qquad \textbf{(i)}$$

In the horizontal (x) direction, the sum of the forces is

$$\Sigma F_x = 0$$
$$F_{Hx} - F_{Tx} = 0. \qquad \textbf{(ii)}$$

For the torque equation, we choose the axis at the point where $\vec{F}_T$ and $M\vec{g}$ act (so our equation then contains only one unknown, F_{Hy}). We choose torques that tend to rotate the beam counterclockwise as positive. The weight mg of the (uniform) beam acts at its center, so we have

$$\Sigma \tau = 0$$
$$-(F_{Hy})(2.20\text{ m}) + mg(1.10\text{ m}) = 0.$$

We solve for F_{Hy}:

$$F_{Hy} = \left(\frac{1.10\text{ m}}{2.20\text{ m}}\right)mg = (0.500)(25.0\text{ kg})(9.80\text{ m/s}^2) = 123\text{ N}. \qquad \textbf{(iii)}$$

Next, since the tension $\vec{F}_T$ in the cable acts along the cable ($\theta = 30.0°$), we see from Fig. 9–10 that $\tan\theta = F_{Ty}/F_{Tx}$, or

$$F_{Ty} = F_{Tx}\tan\theta = F_{Tx}(\tan 30.0°) = 0.577\,F_{Tx}. \qquad \textbf{(iv)}$$

Equation (i) above gives

$$F_{Ty} = (m + M)g - F_{Hy} = (53.0\text{ kg})(9.80\text{ m/s}^2) - 123\text{ N} = 396\text{ N};$$

Equations (iv) and (ii) give

$$F_{Tx} = F_{Ty}/0.577 = 687\text{ N};$$
$$F_{Hx} = F_{Tx} = 687\text{ N}.$$

The components of $\vec{F}_H$ are $F_{Hy} = 123$ N and $F_{Hx} = 687$ N. The tension in the wire is $F_T = \sqrt{F_{Tx}^2 + F_{Ty}^2} = 793$ N.

Alternate Solution Let us see the effect of choosing a different axis for calculating torques, such as an axis through the hinge. Then the lever arm