EXAMPLE 8–16 ESTIMATE **Star collapse.** Astronomers often detect stars that are rotating extremely rapidly, known as neutron stars. These stars are believed to have formed from the inner core of a larger star that collapsed, due to its own gravitation, to a star of very small radius and very high density. Before collapse, suppose the core of such a star is the size of our Sun $(R \approx 7 \times 10^5\,\mathrm{km})$ with mass 2.0 times as great as the Sun, and is rotating at a speed of 1.0 revolution every 10 days. If it were to undergo gravitational collapse to a neutron star of radius 10 km, what would its rotation speed be? Assume the star is a uniform sphere at all times.

PHYSICS APPLIED
Neutron star

APPROACH The star is isolated (no external forces), so we can use conservation of angular momentum for this process.

SOLUTION From conservation of angular momentum,

$$I_1\omega_1 = I_2\omega_2$$

where the subscripts 1 and 2 refer to initial (normal star) and final (neutron star), respectively. Then, assuming no mass is lost in the process,

$$\omega_2 = \left(\frac{I_1}{I_2}\right)\omega_1 = \left(\frac{\frac{2}{5}M_1R_1^2}{\frac{2}{5}M_2R_2^2}\right)\omega_1 = \frac{R_1^2}{R_2^2}\,\omega_1.$$

The frequency $f = \omega/2\pi$, so

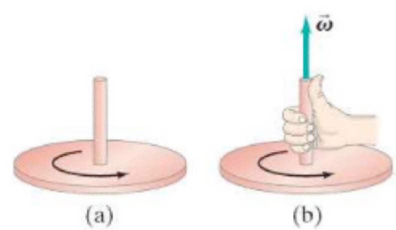
$$f_2 = \frac{\omega_2}{2\pi} = \frac{R_1^2}{R_2^2}f_1$$

$$= \left(\frac{7 \times 10^5\,\mathrm{km}}{10\,\mathrm{km}}\right)^2\left(\frac{1.0\,\mathrm{rev}}{10\,\mathrm{d}(24\,\mathrm{h/d})(3600\,\mathrm{s/h})}\right) \approx 6 \times 10^3\,\mathrm{rev/s}.$$

* 8–9 Vector Nature of Angular Quantities

Up to now we have considered only the magnitudes of angular quantities such as ω, α, and L. But they have a vector aspect too, and now we consider the directions. In fact, we have to *define* the directions for rotational quantities, and we take first the angular velocity, $\vec{\boldsymbol{\omega}}$.

FIGURE 8–31 (a) Rotating wheel. (b) Right-hand rule for obtaining the direction of $\vec{\boldsymbol{\omega}}$.



Consider the rotating wheel shown in Fig. 8–31a. The linear velocities of different particles of the wheel point in all different directions. The only unique direction in space associated with the rotation is along the axis of rotation, perpendicular to the actual motion. We therefore choose the axis of rotation to be the direction of the angular velocity vector, $\vec{\boldsymbol{\omega}}$. Actually, there is still an ambiguity since $\vec{\boldsymbol{\omega}}$ could point in either direction along the axis of rotation (up or down in Fig. 8–31a). The convention we use, called **the right-hand rule**, is the following: When the fingers of the right hand are curled around the rotation axis and point in the direction of the rotation, then the thumb points in the direction of $\vec{\boldsymbol{\omega}}$. This is shown in Fig. 8–31b. Note that $\vec{\boldsymbol{\omega}}$ points in the direction a right-handed screw would move when turned in the direction of rotation. Thus, if the rotation of the wheel in Fig. 8–31b is counterclockwise, the direction of $\vec{\boldsymbol{\omega}}$ is upward. If the wheel rotates clockwise, then $\vec{\boldsymbol{\omega}}$ points in the opposite direction, downward. Note that no part of the rotating object moves in the direction of $\vec{\boldsymbol{\omega}}$.

Right-hand rule

If the axis of rotation is fixed, then $\vec{\boldsymbol{\omega}}$ can change only in magnitude. Thus $\vec{\boldsymbol{\alpha}} = \Delta\vec{\boldsymbol{\omega}}/\Delta t$ must also point along the axis of rotation. If the rotation is counterclockwise as in Fig. 8–31a, and if the magnitude ω is increasing, then $\vec{\boldsymbol{\alpha}}$ points upward; but if ω is decreasing (the wheel is slowing down), $\vec{\boldsymbol{\alpha}}$ points downward. If the rotation is clockwise, $\vec{\boldsymbol{\alpha}}$ will point downward if ω is increasing, and point upward if ω is decreasing.