

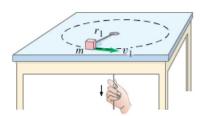
FIGURE 8–28 A skater doing a spin on ice, illustrating conservation of angular momentum. In (a), I is large and ω is small; in (b), I is smaller so ω is larger.



FIGURE 8-29 A diver rotates faster when arms and legs are tucked in than when they are outstretched. Angular momentum is conserved.



FIGURE 8-30 Example 8-15.



The parts of the object may alter their positions relative to one another, so I changes. But then ω changes as well to ensure that the product $I\omega$ remains constant.

Many interesting phenomena can be understood on the basis of conservation of angular momentum. Consider a skater doing a spin on the tips of her skates, Fig. 8–28. She rotates at a relatively low speed when her arms are outstretched; when she brings her arms in close to her body, she suddenly spins much faster. From the definition of moment of inertia, $I = \sum mr^2$, it is clear that when she pulls her arms in closer to the axis of rotation, r is reduced for the arms, so her moment of inertia is reduced. Since the angular momentum $I\omega$ remains constant (we ignore the small torque due to friction), if I decreases, then the angular velocity ω must increase. If the skater reduces her moment of inertia by a factor of 2, she will then rotate with twice the angular velocity.

EXERCISE C When a spinning figure skater pulls in her arms, her moment of inertia decreases; to conserve angular momentum, her angular velocity increases. Does her rotational kinetic energy also increase? If so, where does the energy come from?

A similar example is the diver shown in Fig. 8–29. The push as she leaves the board gives her an initial angular momentum about her center of mass. When she curls herself into the tuck position, she rotates quickly one or more times. She then stretches out again, increasing her moment of inertia which reduces the angular velocity to a small value, and then she enters the water. The change in moment of inertia from the straight position to the tuck position can be a factor of as much as $3\frac{1}{2}$.

Note that for angular momentum to be conserved, the net torque must be zero, but the net force does not necessarily have to be zero. The net force on the diver in Fig. 8–29, for example, is not zero (gravity is acting), but the net torque on her is zero because the force of gravity acts at her center of mass.

EXAMPLE 8–15 Object rotating on a string of changing length. A small mass m attached to the end of a string revolves in a circle on a frictionless tabletop. The other end of the string passes through a hole in the table (Fig. 8–30). Initially, the mass revolves with a speed $v_1 = 2.4 \,\mathrm{m/s}$ in a circle of radius $r_1 = 0.80 \,\mathrm{m}$. The string is then pulled slowly through the hole so that the radius is reduced to $r_2 = 0.48 \,\mathrm{m}$. What is the speed, v_2 , of the mass now?

APPROACH There is no net torque on the mass *m* because the force exerted by the string to keep it moving in a circle is exerted toward the axis; hence the lever arm is zero. We can thus apply conservation of angular momentum.

SOLUTION Conservation of angular momentum gives

$$I_1 \omega_1 = I_2 \omega_2.$$

Our small mass is essentially a particle whose moment of inertia about the hole is $I = mr^2$ (Section 8–5, Eq. 8–11), so we have

$$mr_1^2\omega_1=mr_2^2\omega_2,$$

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$$\omega_2 = \omega_1 \left(\frac{r_1^2}{r_2^2} \right).$$

Then, since $v = r\omega$, we can write

$$v_2 = r_2 \omega_2 = r_2 \omega_1 \left(\frac{r_1^2}{r_2^2}\right) = r_2 \frac{v_1}{r_1} \left(\frac{r_1^2}{r_2^2}\right) = v_1 \frac{r_1}{r_2}$$
$$= (2.4 \text{ m/s}) \left(\frac{0.80 \text{ m}}{0.48 \text{ m}}\right) = 4.0 \text{ m/s}.$$

The speed increases as the radius decreases.

EXERCISE D The speed of mass m in Example 8–15 increased, so its kinetic energy increased. Where did the energy come from?