

The wheel has rotated through a small angle  $\Delta\theta = \Delta l/r$  (Eq. 8-1). Hence

$$W = F\Delta l = Fr\Delta\theta.$$

Since  $\tau = rF$ , then

$$W = \tau\Delta\theta \quad (8-17) \quad \text{Work done by torque}$$

is the work done by the torque  $\tau$  when rotating the wheel through an angle  $\Delta\theta$ . Finally, power  $P$  is the rate work is done:  $P = W/\Delta t = \tau\Delta\theta/\Delta t = \tau\omega$ .

## 8-8 Angular Momentum and Its Conservation

Throughout this Chapter we have seen that if we use the appropriate angular variables, the kinematic and dynamic equations for rotational motion are analogous to those for ordinary linear motion. We saw in the previous Section, for example, that rotational kinetic energy can be written as  $\frac{1}{2}I\omega^2$ , which is analogous to the translational kinetic energy,  $\frac{1}{2}mv^2$ . In like manner, the linear momentum,  $p = mv$ , has a rotational analog. It is called **angular momentum**,  $L$ . For an object rotating about a fixed axis, it is defined as

$$L = I\omega, \quad (8-18) \quad \text{Angular momentum}$$

where  $I$  is the moment of inertia and  $\omega$  is the angular velocity about the axis of rotation. The SI units for  $L$  are  $\text{kg} \cdot \text{m}^2/\text{s}$ , which has no special name.

We saw in Chapter 7 (Section 7-1) that Newton's second law can be written not only as  $\Sigma F = ma$  but also more generally in terms of momentum (Eq. 7-2),  $\Sigma F = \Delta p/\Delta t$ . In a similar way, the rotational equivalent of Newton's second law, which we saw in Eq. 8-14 can be written as  $\Sigma \tau = I\alpha$ , can also be written in terms of angular momentum:

$$\Sigma \tau = \frac{\Delta L}{\Delta t}, \quad (8-19)$$

NEWTON'S SECOND LAW  
FOR ROTATION

where  $\Sigma \tau$  is the net torque acting to rotate the object, and  $\Delta L$  is the change in angular momentum in a time interval  $\Delta t$ . Equation 8-14,  $\Sigma \tau = I\alpha$ , is a special case of Eq. 8-19 when the moment of inertia is constant. This can be seen as follows. If an object has angular velocity  $\omega_0$  at time  $t = 0$ , and angular velocity  $\omega$  after a time interval  $\Delta t$ , then its angular acceleration (Eq. 8-3) is

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t}.$$

Then from Eq. 8-19, we have

$$\Sigma \tau = \frac{\Delta L}{\Delta t} = \frac{I\omega - I\omega_0}{\Delta t} = \frac{I(\omega - \omega_0)}{\Delta t} = I \frac{\Delta\omega}{\Delta t} = I\alpha,$$

which is Eq. 8-14.

Angular momentum is an important concept in physics because, under certain conditions, it is a conserved quantity. We can see from Eq. 8-19 that if the net torque  $\Sigma \tau$  on an object is zero, then  $\Delta L/\Delta t$  equals zero. That is,  $L$  does not change. This is the **law of conservation of angular momentum** for a rotating object:

**The total angular momentum of a rotating object remains constant if the net torque acting on it is zero.**

CONSERVATION OF  
ANGULAR MOMENTUM

The law of conservation of angular momentum is one of the great conservation laws of physics, along with energy and linear momentum.

When there is zero net torque acting on an object, and the object is rotating about a fixed axis or about an axis through its center of mass whose direction doesn't change, we can write

$$I\omega = I_0\omega_0 = \text{constant}.$$

$I_0$  and  $\omega_0$  are the moment of inertia and angular velocity, respectively, about that axis at some initial time ( $t = 0$ ), and  $I$  and  $\omega$  are their values at some other time.