CONCEPTUAL EXAMPLE 8–14 **Who's fastest?** Several objects roll without slipping down an incline of vertical height H, all starting from rest at the same moment. The objects are a thin hoop (or a plain wedding band), a spherical marble, a solid cylinder (a D-cell battery), and an empty soup can. In addition, a greased box slides down without friction. In what order do they reach the bottom of the incline?

RESPONSE The sliding box wins because the potential energy loss (MgH) is transformed completely into translational KE for the box, whereas for rolling objects the initial PE is shared between translational and rotational kinetic energies, and so their linear speed is less. For each of the rolling objects we can state that the loss in potential energy equals the increase in kinetic energy:

$$MgH = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I_{CM}\omega^2.$$

For all our rolling objects, the moment of inertia I_{CM} is a numerical factor times the mass M and the radius R^2 (Fig. 8–21). The mass M is in each term, so the translational speed v doesn't depend on M; nor does it depend on the radius R since $\omega = v/R$, so R^2 cancels out for all the rolling objects, just as in Example 8–13. Thus the speed v at the bottom depends only on that numerical factor in I_{CM} which expresses how the mass is distributed. The hoop, with all its mass concentrated at radius R $(I_{CM} = MR^2)$, has the largest moment of inertia; hence it will have the lowest speed and will arrive at the bottom behind the D-cell $\left(I_{CM} = \tfrac{1}{2}MR^2\right)$, which in turn will be behind the marble $\left(I_{CM} = \tfrac{2}{5}MR^2\right)$. The empty can, which is mainly a hoop plus a small disk, has most of its mass concentrated at R; so it will be a bit faster than the pure hoop but slower than the D-cell. See Fig. 8–25.

NOTE As in Example 8–13, the speed at the bottom does not depend on the object's mass M or radius R, but only on its shape (and the height of the hill H).

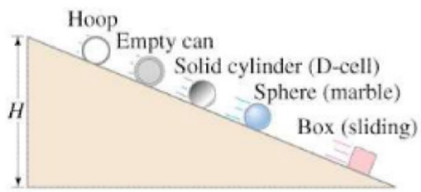
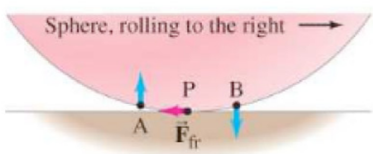


FIGURE 8–25 Example 8–14.

FIGURE 8–26 A sphere rolling to the right on a plane surface. The point in contact with the ground at any moment, point P, is momentarily at rest. Point A to the left of P is moving nearly vertically upward at the instant shown, and point B to the right is moving nearly vertically downward. An instant later, point B will touch the plane and be at rest momentarily. Thus no work is done by the force of static friction.



CAUTION

Rolling objects go slower than sliding objects because of rotational KE, not because of friction

If there had been little or no static friction between the rolling objects and the plane in these Examples, the round objects would have slid rather than rolled, or a combination of both. Static friction must be present to make a round object roll. We did not need to take friction into account in the energy equation because it is *static* friction and does no work—the point of contact of the sphere at each instant does not slide, but moves perpendicular to the plane (first down and then up as shown in Fig. 8–26) as the sphere rolls. Thus, no work is done by the static friction force because the force and the motion (displacement) are perpendicular. The reason the rolling objects in Examples 8–13 and 8–14 move down the slope more slowly than if they were sliding is *not* because friction is doing work. Rather, it is because some of the gravitional PE is converted to rotational KE, leaving less for the translational KE.

Work Done by Torque

The work done on an object rotating about a fixed axis, such as the pulleys in Figs. 8–22 and 8–23, can be written using angular quantities. As shown in Fig. 8–27, a force F exerting a torque $\tau = rF$ on a wheel does work $W = F\Delta l$ in rotating the wheel a small distance Δl at the point of application of $\vec{\mathbf{F}}$.

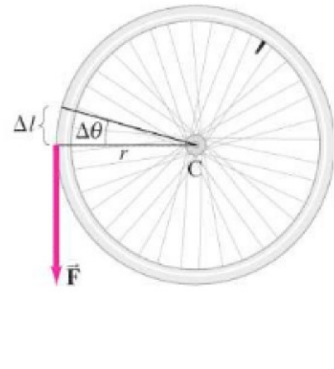


FIGURE 8–27 Torque $\tau = rF$ does work when rotating a wheel equal to $W = F\Delta l = Fr\Delta\theta = \tau\Delta\theta$.