Consider any rigid rotating object as made up of many tiny particles, each of mass m. If we let r represent the distance of any one particle from the axis of rotation, then its linear velocity is $v = r\omega$. The total kinetic energy of the whole object will be the sum of the kinetic energies of all its particles:

$$\text{KE} = \Sigma\left(\tfrac{1}{2}mv^2\right) = \Sigma\left(\tfrac{1}{2}mr^2\omega^2\right)$$
$$= \tfrac{1}{2}\Sigma(mr^2)\omega^2.$$

We have factored out the $\frac{1}{2}$ and the ω^2 since they are the same for every particle of a rigid object. Since $\Sigma mr^2 = I$, the moment of inertia, we see that the kinetic energy of a rigid rotating object is, as expected,

$$\text{rotational KE} = \tfrac{1}{2}I\omega^2. \qquad \textbf{(8–15)}$$

Rotational KE

The units are joules, as with all other forms of energy.

An object that rotates while its center of mass (CM) undergoes translational motion will have both translational and rotational kinetic energies. Equation 8–15 gives the rotational kinetic energy if the rotation axis is fixed. If the object is moving (such as a wheel rolling down a hill), this equation is still valid as long as the rotation axis is fixed in direction. Then the total kinetic energy is

$$\text{KE} = \tfrac{1}{2}Mv_{\text{CM}}^2 + \tfrac{1}{2}I_{\text{CM}}\,\omega^2, \qquad \textbf{(8–16)}$$

Total KE (translation + rotation)

where v_{CM} is the linear velocity of the center of mass, I_{CM} is the moment of inertia about an axis through the center of mass, ω is the angular velocity about this axis, and M is the total mass of the object.

EXAMPLE 8–13 **Sphere rolling down an incline.** What will be the speed of a solid sphere of mass M and radius R when it reaches the bottom of an incline if it starts from rest at a vertical height H and rolls without slipping? See Fig. 8–24. (Assume plenty of static friction, which does no work, so no slipping takes place.) Compare your result to that for an object *sliding* down a frictionless incline.

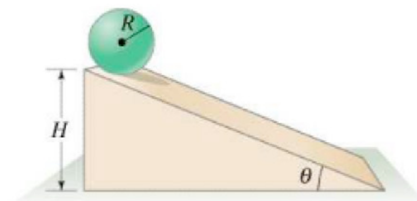

FIGURE 8–24 A sphere rolling down a hill has both translational and rotational kinetic energy. Example 8–13.

APPROACH We use the law of conservation of energy with gravitational potential energy, now including rotational kinetic energy as well as translational KE.

SOLUTION The total energy at any point a vertical distance y above the base of the incline is

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I_{\text{CM}}\,\omega^2 + Mgy,$$

where v is the speed of the center of mass, and Mgy is the gravitational PE. Applying conservation of energy, we equate the total energy at the top ($y = H, v = 0, \omega = 0$) to the total energy at the bottom ($y = 0$):

$$0 + 0 + MgH = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I_{\text{CM}}\omega^2 + 0.$$

PROBLEM SOLVING

Rotational energy adds to other forms of energy to get the total energy which is conserved

The moment of inertia of a solid sphere about an axis through its center of mass is $I_{\text{CM}} = \frac{2}{5}MR^2$, Fig. 8–21e. Since the sphere rolls without slipping, we have $\omega = v/R$ (recall Fig. 8–8). Hence

$$MgH = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}MR^2\right)\left(\frac{v^2}{R^2}\right).$$

Canceling the M's and R's, we obtain

$$\left(\tfrac{1}{2} + \tfrac{1}{5}\right)v^2 = gH$$

or

$$v = \sqrt{\tfrac{10}{7}gH}.$$

We can compare this result for the speed of a rolling sphere to that for an object sliding down a plane without rotating and without friction, $\frac{1}{2}mv^2 = mgH$ (see our energy equation above, removing the rotational term). Then $v = \sqrt{2gH}$, which is greater than our result. An object sliding without friction or rotation transforms its initial potential energy entirely into translational KE (none into rotational KE), so the speed of its center of mass is greater.

NOTE Our result for the rolling sphere shows (perhaps surprisingly) that v is independent of both the mass M and the radius R of the sphere.