PROBLEM SOLVING Rotational Motion

1. As always, draw a clear and complete **diagram**.
2. Choose the object or objects that will be the **system** to be studied.
3. Draw a **free-body diagram** for the object under consideration (or for each object, if more than one), showing only (and all) the forces acting on that object and exactly where they act, so you can determine the torque due to each. Gravity acts at the CG of the object (Section 7–8).
4. Identify the axis of rotation and determine the **torques** about it. Choose positive and negative directions of rotation (counterclockwise and clockwise), and assign the correct sign to each torque.
5. Apply **Newton's second law for rotation**, $\Sigma\tau = I\alpha$. If the moment of inertia is not given, and it is not the unknown sought, you need to determine it first. Use consistent units, which in SI are: α in $\mathrm{rad/s^2}$; τ in $\mathrm{m\cdot N}$; and I in $\mathrm{kg\cdot m^2}$.
6. Also apply **Newton's second law for translation**, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, and **other** laws or principles as needed.
7. **Solve** the resulting equation(s) for the unknown(s).
8. Do a rough **estimate** to determine if your answer is reasonable.

EXAMPLE 8–11 **A heavy pulley.** A 15.0-N force (represented by $\vec{\mathbf{F}}_{\mathrm{T}}$) is applied to a cord wrapped around a pulley of mass $M = 4.00\,\mathrm{kg}$ and radius $R = 33.0\,\mathrm{cm}$, Fig. 8–22. The pulley accelerates uniformly from rest to an angular speed of 30.0 rad/s in 3.00 s. If there is a frictional torque $\tau_{\mathrm{fr}} = 1.10\,\mathrm{m\cdot N}$ at the axle, determine the moment of inertia of the pulley. The pulley rotates about its center.

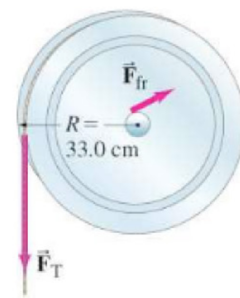

FIGURE 8–22 Example 8–11.

APPROACH We follow the steps of the Problem Solving Box explicitly.

SOLUTION

1. **Draw a diagram.** The pulley and the attached cord are shown in Fig. 8–22.
2. **Choose the system**: the pulley.
3. **Draw a free-body diagram.** The force that the cord exerts on the pulley is shown as $\vec{\mathbf{F}}_{\mathrm{T}}$ in Fig. 8–22. The friction force is also shown, but we are given only its torque. Two other forces could be included in the diagram: the force of gravity mg down and whatever force keeps the axle in place. They do not contribute to the torque (their lever arms are zero) and so are not shown.
4. **Determine the torques.** The cord exerts a force $\vec{\mathbf{F}}_{\mathrm{T}}$ that acts at the edge of the pulley, so its lever arm is R. The torque exerted by the cord equals RF_{T} and is counterclockwise, which we choose to be positive. The frictional torque is given as $\tau_{\mathrm{fr}} = 1.10\,\mathrm{m\cdot N}$; it opposes the motion and is negative.
5. **Apply Newton's second law for rotation.** The net torque is
$$\begin{aligned}\Sigma\tau &= RF_{\mathrm{T}} - \tau_{\mathrm{fr}}\\ &= (0.330\,\mathrm{m})(15.0\,\mathrm{N}) - 1.10\,\mathrm{m\cdot N} = 3.85\,\mathrm{m\cdot N}.\end{aligned}$$
The angular acceleration α is found from the given data that it takes 3.0 s to accelerate the pulley from rest to $\omega = 30.0\,\mathrm{rad/s}$:
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{30.0\,\mathrm{rad/s} - 0}{3.00\,\mathrm{s}} = 10.0\,\mathrm{rad/s^2}.$$
We can now solve for I in Newton's second law (see step 7).
6. **Other calculations**: None needed.
7. **Solve for unknowns.** We solve for I in Newton's second law for rotation, $\Sigma\tau = I\alpha$, and insert our values for $\Sigma\tau$ and α:
$$I = \frac{\Sigma\tau}{\alpha} = \frac{3.85\,\mathrm{m\cdot N}}{10.0\,\mathrm{rad/s^2}} = 0.385\,\mathrm{kg\cdot m^2}.$$
8. **Do a rough estimate.** We can do a rough estimate of the moment of inertia by assuming the pulley is a uniform cylinder and using Fig. 8–21c:
$$I = \tfrac{1}{2}MR^2 = \tfrac{1}{2}(4.00\,\mathrm{kg})(0.330\,\mathrm{m})^2 = 0.218\,\mathrm{kg\cdot m^2}.$$
This is the same order of magnitude as our result, but numerically somewhat less. This makes sense, though, because a pulley is not usually a uniform cylinder but instead has more of its mass concentrated toward the outside edge. Such a pulley would be expected to have a greater moment of inertia than a solid cylinder of equal mass; a thin hoop, Fig. 8–21a, ought to have a greater I than our pulley, and indeed it does: $I = MR^2 = 0.436\,\mathrm{kg\cdot m^2}$.

Usefulness and power of rough estimates