rotation. If we assign each particle a number $(1, 2, 3, \ldots)$, then $\Sigma mr^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots$. This quantity is called the **moment of inertia** (or *rotational inertia*) $I$ of the object:

$$I = \Sigma mr^2 = m_1 r_1^2 + m_2 r_2^2 + \cdots. \tag{8–13}$$

*Moment of inertia*

Combining Eqs. 8–12 and 8–13, we can write

$$\Sigma \tau = I\alpha. \tag{8–14}$$

*NEWTON'S SECOND LAW FOR ROTATION*

This is the rotational equivalent of Newton's second law. It is valid for the rotation of a rigid object about a fixed axis.†

We see that the moment of inertia, $I$, which is a measure of the rotational inertia of an object, plays the same role for rotational motion that mass does for translational motion. As can be seen from Eq. 8–13, the rotational inertia of an object depends not only on its mass, but also on how that mass is distributed with respect to the axis. For example, a large-diameter cylinder will have greater rotational inertia than one of equal mass but smaller diameter (and therefore greater length), Fig. 8–19. The former will be harder to start rotating, and harder to stop. When the mass is concentrated farther from the axis of rotation, the rotational inertia is greater. For rotational motion, the mass of an object *cannot* be considered as concentrated at its center of mass.

**CAUTION**

*Mass can* not *be considered concentrated at* CM *for rotational motion*

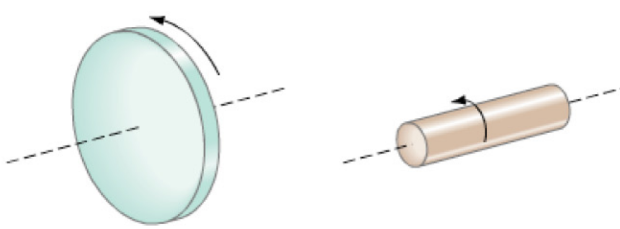


**FIGURE 8–19** A large-diameter cylinder has greater rotational inertia than one of smaller diameter but equal mass.

**EXAMPLE 8–10** **Two weights on a bar: different axis, different *I*.** Two small "weights," of mass 5.0 kg and 7.0 kg, are mounted 4.0 m apart on a light rod (whose mass can be ignored), as shown in Fig. 8–20. Calculate the moment of inertia of the system (*a*) when rotated about an axis halfway between the weights, Fig. 8–20a, and (*b*) when rotated about an axis 0.50 m to the left of the 5.0-kg mass (Fig. 8–20b).

**APPROACH** In each case, the moment of inertia of the system is found by summing over the two parts using Eq. 8–13.

**SOLUTION** (*a*) Both weights are the same distance, 2.0 m, from the axis of rotation. Thus

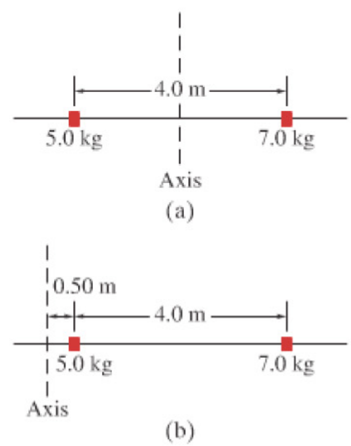
$$\begin{aligned} I = \Sigma mr^2 &= (5.0\,\text{kg})(2.0\,\text{m})^2 + (7.0\,\text{kg})(2.0\,\text{m})^2 \\ &= 20\,\text{kg}\cdot\text{m}^2 + 28\,\text{kg}\cdot\text{m}^2 = 48\,\text{kg}\cdot\text{m}^2. \end{aligned}$$

(*b*) The 5.0-kg mass is now 0.50 m from the axis, and the 7.0-kg mass is 4.50 m from the axis. Then

$$\begin{aligned} I = \Sigma mr^2 &= (5.0\,\text{kg})(0.50\,\text{m})^2 + (7.0\,\text{kg})(4.5\,\text{m})^2 \\ &= 1.3\,\text{kg}\cdot\text{m}^2 + 142\,\text{kg}\cdot\text{m}^2 = 143\,\text{kg}\cdot\text{m}^2. \end{aligned}$$

**NOTE** This Example illustrates two important points. First, the moment of inertia of a given system is different for different axes of rotation. Second, we see in part (*b*) that mass close to the axis of rotation contributes little to the total moment of inertia; here, the 5.0-kg object contributed less than 1% to the total.

**FIGURE 8–20** Example 8–10: calculating the moment of inertia.




**CAUTION**

*I depends on axis of rotation and on distribution of mass*

†Equation 8–14 is also valid when the object is translating with acceleration, as long as $I$ and $\alpha$ are calculated about the center of mass of the object, and the rotation axis through the CM doesn't change direction.