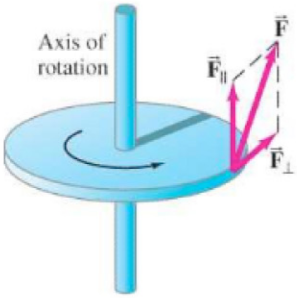


FIGURE 8–17 Only the component of $\vec{\mathbf{F}}$ that acts in the plane perpendicular to the rotation axis, $\vec{\mathbf{F}}_{\perp}$, acts to turn the wheel about the axis. The component parallel to the axis, $\vec{\mathbf{F}}_{\parallel}$, would tend to move the axis itself, which we assume is held fixed.

* Forces that Act to Tilt the Axis

We are considering only rotation about a fixed axis, and so we consider only forces that act in a plane perpendicular to the axis of rotation. If there is a force (or component of a force) acting parallel to the axis of rotation, it will tend to tilt the axis of rotation—the component $\vec{\mathbf{F}}_{\parallel}$ in Fig. 8–17 is an example. Since we are assuming the axis remains fixed in direction, either there can be no such forces or else the axis must be mounted in bearings or hinges that hold the axis fixed. Thus, only a force, or component of a force ($\mathbf{F}_{\perp}$ in Fig. 8–17), in a plane perpendicular to the axis will give rise to rotation about the axis, and it is only these that we consider.

8–5 Rotational Dynamics; Torque and Rotational Inertia

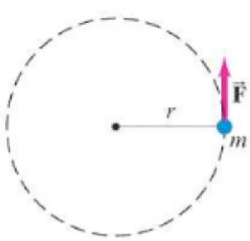
We have discussed that the angular acceleration α of a rotating object is proportional to the net torque τ applied to it:

$$\alpha \propto \Sigma\tau,$$

where we write $\Sigma\tau$ to remind us[†] that it is the *net* torque (sum of all torques acting on the object) that is proportional to α. This corresponds to Newton's second law for translational motion, $a \propto \Sigma F$, but here torque has taken the place of force, and, correspondingly, the angular acceleration α takes the place of the linear acceleration a. In the linear case, the acceleration is not only proportional to the net force, but it is also inversely proportional to the inertia of the object, which we call its mass, m. Thus we could write $a = \Sigma F/m$. But what plays the role of mass for the rotational case? That is what we now set out to determine. At the same time, we will see that the relation $\alpha \propto \Sigma\tau$ follows directly from Newton's second law, $\Sigma F = ma$.

We first consider a very simple case: a particle of mass m rotating in a circle of radius r at the end of a string or rod whose mass we can ignore compared to m (Fig. 8–18), and we assume a single force F acts on m as shown. The torque that gives rise to the angular acceleration is $\tau = rF$. If we use Newton's second law for linear quantities, $\Sigma F = ma$, and Eq. 8–5 relating the angular acceleration to the tangential linear acceleration, $a_{\tan} = r\alpha$, then we have

FIGURE 8–18 A mass m rotating in a circle of radius r about a fixed point.



$$\begin{aligned} F &= ma \\ &= mr\alpha. \end{aligned}$$

When we multiply both sides of this equation by r, we find that the torque $\tau = rF$ is given by

$$\tau = mr^2\alpha. \qquad \text{[single particle]} \quad \textbf{(8–11)}$$

Here at last we have a direct relation between the angular acceleration and the applied torque τ. The quantity mr^2 represents the *rotational inertia* of the particle and is called its *moment of inertia.*

Now let us consider a rotating rigid object, such as a wheel rotating about an axis through its center, which could be an axle. We can think of the wheel as consisting of many particles located at various distances from the axis of rotation. We can apply Eq. 8–11 to each particle of the object, and then sum over all the particles. The sum of the various torques is just the total torque, $\Sigma\tau$, so we obtain:

$$\Sigma\tau = \left(\Sigma mr^2\right)\alpha \qquad \textbf{(8–12)}$$

where we factored out α because it is the same for all the particles of the object. The sum Σmr^2 represents the sum of the masses of each particle in the object multiplied by the square of the distance of that particle from the axis of

[†]Recall from Chapter 4 that Σ (Greek letter sigma) means "sum of."