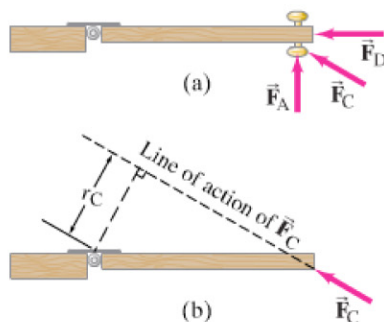


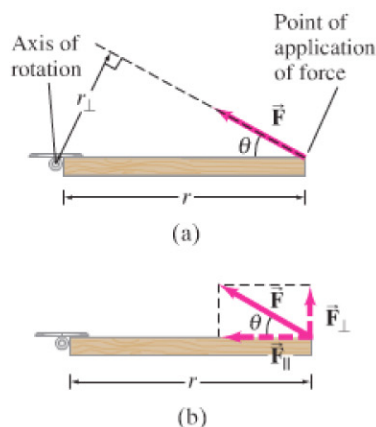
### Torque defined



**FIGURE 8-12** (a) Forces acting at different angles at the doorknob. (b) The lever arm is defined as the perpendicular distance from the axis of rotation (the hinge) to the line of action of the force.

**FIGURE 8-13**

Torque =  $r_{\perp}F = rF_{\perp}$ .



### Magnitude of a torque

The angular acceleration, then, is proportional to the product of the *force times the lever arm*. This product is called the *moment of the force* about the axis, or, more commonly, it is called the **torque**, and is represented by  $\tau$  (Greek lowercase letter tau). Thus, the angular acceleration  $\alpha$  of an object is directly proportional to the net applied torque  $\tau$ :

$$\alpha \propto \tau,$$

and we see that it is torque that gives rise to angular acceleration. This is the rotational analog of Newton's second law for linear motion,  $a \propto F$ .

We defined the lever arm as the *perpendicular* distance from the axis of rotation to the line of action of the force—that is, the distance which is perpendicular both to the axis of rotation and to an imaginary line drawn along the direction of the force. We do this to take into account the effect of forces acting at an angle. It is clear that a force applied at an angle, such as  $\vec{F}_C$  in Fig. 8-12, will be less effective than the same magnitude force applied perpendicular to the door, such as  $\vec{F}_A$  (Fig. 8-12a). And if you push on the end of the door so that the force is directed at the hinge (the axis of rotation), as indicated by  $\vec{F}_D$ , the door will not rotate at all.

The lever arm for a force such as  $\vec{F}_C$  is found by drawing a line along the direction of  $\vec{F}_C$  (this is the “line of action” of  $\vec{F}_C$ ). Then we draw another line, perpendicular to this line of action, that goes to the axis of rotation and is perpendicular also to it. The length of this second line is the lever arm for  $\vec{F}_C$  and is labeled  $r_C$  in Fig. 8-12b. The lever arm is perpendicular both to the line of action of the force and, at its other end, perpendicular to the rotation axis.

The magnitude of the torque associated with  $\vec{F}_C$  is then  $r_C F_C$ . This short lever arm  $r_C$  and the corresponding smaller torque associated with  $\vec{F}_C$  is consistent with the observation that  $\vec{F}_C$  is less effective in accelerating the door than is  $\vec{F}_A$ . When the lever arm is defined in this way, experiment shows that the relation  $\alpha \propto \tau$  is valid in general. Notice in Fig. 8-12 that the line of action of the force  $\vec{F}_D$  passes through the hinge, and hence its lever arm is zero. Consequently, zero torque is associated with  $\vec{F}_D$  and it gives rise to no angular acceleration, in accord with everyday experience.

In general, then, we can write the magnitude of the torque about a given axis as

$$\tau = r_{\perp} F, \quad (8-10a)$$

where  $r_{\perp}$  is the lever arm, and the perpendicular symbol ( $\perp$ ) reminds us that we must use the distance from the axis of rotation that is perpendicular to the line of action of the force (Fig. 8-13a).

An equivalent way of determining the torque associated with a force is to resolve the force into components parallel and perpendicular to the line that connects the axis to the point of application of the force, as shown in Fig. 8-13b. The component  $F_{\parallel}$  exerts no torque since it is directed at the rotation axis (its moment arm is zero). Hence the torque will be equal to  $F_{\perp}$  times the distance  $r$  from the axis to the point of application of the force:

$$\tau = r F_{\perp}. \quad (8-10b)$$

That this gives the same result as Eq. 8-10a can be seen from the relations  $F_{\perp} = F \sin \theta$  and  $r_{\perp} = r \sin \theta$ . [Note that  $\theta$  is the angle between the directions of  $\vec{F}$  and  $r$  (radial line from the axis to the point where  $\vec{F}$  acts)]. So

$$\tau = r F \sin \theta \quad (8-10c)$$

in either case. We can use any of Eqs. 8-10 to calculate the torque, whichever is easiest.

Since torque is a distance times a force, it is measured in units of  $\text{m} \cdot \text{N}$  in SI units,<sup>†</sup>  $\text{cm} \cdot \text{dyne}$  in the cgs system, and  $\text{ft} \cdot \text{lb}$  in the English system.

<sup>†</sup>Note that the units for torque are the same as those for energy. We write the unit for torque here as  $\text{m} \cdot \text{N}$  (in SI) to distinguish it from energy ( $\text{N} \cdot \text{m}$ ) because the two quantities are very different. An obvious difference is that energy is a scalar, whereas torque has a direction and is a vector. The special name *joule* ( $1 \text{ J} = 1 \text{ N} \cdot \text{m}$ ) is used only for energy (and for work), *never* for torque.