

## 8-2 Constant Angular Acceleration

In Chapter 2, we derived the useful kinematic equations (Eqs. 2-11) that relate acceleration, velocity, distance, and time for the special case of uniform linear acceleration. Those equations were derived from the definitions of linear velocity and acceleration, assuming constant acceleration. The definitions of angular velocity and angular acceleration are the same as those for their linear counterparts, except that  $\theta$  has replaced the linear displacement  $x$ ,  $\omega$  has replaced  $v$ , and  $\alpha$  has replaced  $a$ . Therefore, the angular equations for **constant angular acceleration** will be analogous to Eqs. 2-11 with  $x$  replaced by  $\theta$ ,  $v$  by  $\omega$ , and  $a$  by  $\alpha$ , and they can be derived in exactly the same way. We summarize them here, opposite their linear equivalents (we've chosen  $x_0 = 0$ , and  $\theta_0 = 0$  at the initial time  $t = 0$ ):

Angular	Linear		
$\omega = \omega_0 + \alpha t$	$v = v_0 + at$	[constant $\alpha, a$ ] (8-9a)	<i>Kinematic equations</i>
$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$	$x = v_0 t + \frac{1}{2}at^2$	[constant $\alpha, a$ ] (8-9b)	<i>for constant</i>
$\omega^2 = \omega_0^2 + 2\alpha\theta$	$v^2 = v_0^2 + 2ax$	[constant $\alpha, a$ ] (8-9c)	<i>angular acceleration</i>
$\bar{\omega} = \frac{\omega + \omega_0}{2}$	$\bar{v} = \frac{v + v_0}{2}$	[constant $\alpha, a$ ] (8-9d)	$(x_0 = 0, \theta_0 = 0)$

Note that  $\omega_0$  represents the angular velocity at  $t = 0$ , whereas  $\theta$  and  $\omega$  represent the angular position and velocity, respectively, at time  $t$ . Since the angular acceleration is constant,  $\alpha = \bar{\alpha}$ .

**EXAMPLE 8-6 Centrifuge acceleration.** A centrifuge rotor is accelerated from rest to 20,000 rpm in 30 s. (a) What is its average angular acceleration? (b) Through how many revolutions has the centrifuge rotor turned during its acceleration period, assuming constant angular acceleration?

**APPROACH** To determine  $\bar{\alpha} = \Delta\omega/\Delta t$ , we need the initial and final angular velocities. For (b), we use Eqs. 8-9 (recall that one revolution corresponds to  $\theta = 2\pi$  rad).

**SOLUTION** (a) The initial angular velocity is  $\omega = 0$ . The final angular velocity is

$$\omega = 2\pi f = (2\pi \text{ rad/rev}) \frac{(20,000 \text{ rev/min})}{(60 \text{ s/min})} = 2100 \text{ rad/s}.$$

Then, since  $\bar{\alpha} = \Delta\omega/\Delta t$  and  $\Delta t = 30$  s, we have

$$\bar{\alpha} = \frac{\omega - \omega_0}{\Delta t} = \frac{2100 \text{ rad/s} - 0}{30 \text{ s}} = 70 \text{ rad/s}^2.$$

That is, every second the rotor's angular velocity increases by 70 rad/s, or by  $(70/2\pi) = 11$  revolutions per second.

(b) To find  $\theta$  we could use either Eq. 8-9b or 8-9c, or both to check our answer. The former gives

$$\theta = 0 + \frac{1}{2}(70 \text{ rad/s}^2)(30 \text{ s})^2 = 3.15 \times 10^4 \text{ rad},$$

where we have kept an extra digit because this is an intermediate result. To find the total number of revolutions, we divide by  $2\pi$  rad/rev and obtain

$$\frac{3.15 \times 10^4 \text{ rad}}{2\pi \text{ rad/rev}} = 5.0 \times 10^3 \text{ rev}.$$

**NOTE** Let us calculate  $\theta$  using Eq. 8-9c:

$$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(2100 \text{ rad/s})^2 - 0}{2(70 \text{ rad/s}^2)} = 3.15 \times 10^4 \text{ rad}$$

which checks our answer perfectly.